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Valuation of machinery and equipment for industrial properties

Aly Abd Elghaffar Elfar
Iowa State University

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Valuation of machinery and equipment for industrial properties

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Aly Abd Elghaffar Elfar

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INTRODUCTION

> The estimation of the monetary measure of the desirability of ownership of commodities and small properties is accomplished daily in the commercial world, often in an informal and intuitive manner. However, the complex society of the present demands systematic and theoretically correct procedures when consideration is given to the appraisal of enterprises and properties not regularly acquired on the market (1, p. v).

The above quotation is valid today as it was when first written. Value is the end result sought. It is often said after Judge Brandeis that value is a word with many meanings (2). While this may imply confusion and uncertainty, it is an important reminder that the quantity arrived at must, in every situation, be related to the basis upon which it was estimated and the purpose for which it was sought. Marston et al. state concerning value (1):

> The word value in itself is difficult of precise definition and usage. Value is a relative term by which the desirability of ownership of the property in question is stated in terms of other property or money (1, p. 4).

They also state:

> In most uses of the word value as applied to the property there is implied by the author or speaker a sense of worth, a desirability of ownership or possession, or the exchangeability of property as it can be measured in terms of the dollar or other monetary units (1, p. 3).

Bonbright discusses the definitions and the basic concepts of value at length in his treatise on the valuation of property (3). Two basic concepts of value emerged from these discussions, namely, market value and value to the owner. As for market value he states:

> Our own preference, at least in the field of appraisal, is for the previous definition of market value, under which a valuation of property means merely an attempt to estimate the price for which the property could be sold by some stipulated seller to anyone else the conditions of the assumed sale being left for selection by reference to the purpose for which the valuation is being made (3, p. 65).

For the second concept, value to the owner, Bonbright proposes:

> The value of a property to its owner is identical in amount with the adverse value of the entire loss, direct and indirect, that the owner might expect to suffer if he were to be deprived of the property (3, p. 71).

Another author expresses the opinion that:

> Market value is a rather rigidly defined concept. One alternative way of looking into it is to consider it a forecast of most probable selling price. When most probable selling price is estimated, the use pattern in terms of which value is estimated (or selling price is forecast) should be most probable use rather than highest and best use (4, p. 13).

Marston et al. define what they call the fundamental basis of value as:

> The fundamental basis of the value of any specific property is the present worth, to the present owner and to the would-be purchaser, of the probable future services expected from the property during its probable future productive life in service. The future service may be of such character as to bring an annual money return to the owner, as in the case of real estate rented; or the future service may be of value to the owner because of his use of the property, as in the case of food consumed or a house occupied; or the service may be of value to the owner mainly because of personal satisfaction in its ownership, as in the case of jewelry or a painting (1, p. 5).

About the same issue of the present worth, Winfrey wrote:

> The basic principle of the present-worth method is that the worth today of a property unit is no more or no less than the present worth of the future returns from that unit. Operation of the unit should produce, above current operation expenses, an annual operation return sufficient to provide a net investment return and a return for depreciation (5, p. 24).

The present worth or the present value were also set forth by two leading economists, Fisher and Keynes. In 1930, Fisher defined "the rate of return over cost" as:

> that rate which, employed in computing the present worth of all the costs and the present worth of all the returns, will make these two equal (6, p. 168).

Later in 1936, Keynes gave the same thing a different name:

> More precisely, I define the marginal efficiency of capital as being equal to that rate of discount which would make the present value of the series of annuities given by the returns expected from the capital-asset during its life just equal to its supply price (7, p. 135).

The supply price, he explained, is:

> . . . not the market-price at which an asset of the type in question can actually be purchased in the market, but the price which would just induce a manufacturer newly to produce an additional unit of such assets, i.e. what is sometimes called its replacement cost.

Also, about the value of property, Babcock writes:

> It is necessary to emphasize that it is not the income productivity of property which produces its value but rather its anticipated or expected income productivity. The occurrence of the income is subsequent to the evaluation and is therefore anticipated rather than actual. Thus in the development of a valuation theory we commence with the principle that the process of valuation will consist of translating anticipated future income into present capital value. The value of a real property must be computed from its estimated future net income (8, p. 130).

STATEMENT OF THE PROBLEM

The valuation engineer is frequently confronted with the problem of appraising the machinery and equipment as a part of an operating entity. This appraisal could be done for one or more of the following reasons (1, p. 11):

1 - For utility rate making.

2 - To provide information for management.

3 - For ad valorem taxes assessment.

4 - For sale or transfer of a business.

5 - For condemnations.

6 - For settling estates.

7 - To set insurance rates.

8 - In issuing securities and financing purposes.

The interest in this study was first aroused by the situation of assessing taxes for ad valorem tax purposes. In Iowa and some other states, ad valorem taxes are based on market value of the property. For example, the Iowa Code, section 441.21 requires that all real and tangible property subject to taxation shall be valued at its actual value which is defined as the fair and reasonable market value of such property.

In the majority of situations, market value of the machinery and equipment of a going concern cannot be ascertained directly. This may be due to the uniqueness of the property or due to the rarity or nonexistence of market exchanges of comparable property. About this problem, Bonbright says:

> Most of the serious difficulties with the concept of market value or exchange value are encountered when one tries to make use of the concept in assigning a value to property off the market place - to property which is not in process of sale and which may not even be offered for sale. These are the conditions which give rise to the problems of professional appraisal; yet they are the conditions to which the economic theorists have given least attention (3, p. 43).

Leading theorists had suggested ways to deal with the problem of arriving at value. In general, evidences of value are sought, weighed as to appropriateness, and then with the application of judgment value can be estimated. These approaches will be discussed later.

In essense, the problem which will be dealt with here is that of valuation of industrial items or groups of properties whose earnings cannot be earmarked and segregated from the earnings of the enterprise and whose market value cannot be ascertained directly.

It should be pointed out that, in general, the sum of the values of the comprising parts of an enterprise does not necessarily equal the value of the enterprise as a whole (3, p. 80). This disparity may occur when applying the cost approach of valuation which is discussed in a later section. However, the present inquiry is concerned with the valuation of the comprising units or groups of units and not with the value of the enterprise as a whole.

OBJECTIVES

This study is aimed at the problem of valuation of industrial machinery and equipment. The fact that in real life situations the annual earnings and the annual costs of these properties are extremely difficult to ascertain led to the use of methods other than discounting the actual returns expected from the property in the future. To solve this problem, Winfrey developed what is called the condition percent factor defined as "the ratio of the present value of the depreciable property relative to its depreciable value when new" (5, p. 26). This factor is linked to the life of the property and was developed on the premise of uniform annual services derivable from the property.

In this research effort, it is tried to offer a systematic approach which accommodate the situation of declining annual property services. In this perspective, the objectives of the study are:

1 - To develop expressions giving the condition percent factors and estimated values at different ages of property units whose service worths are either declining or uniform through their probable lives.

2 - To develop expressions for the condition percent factors and estimated values at different ages for groups of property composed of units experiencing either declining or uniform service worths through

their service lives.

3 - To gather evidences of life and retirement dispersion of the machinery and equipment of representative Iowa industries necessary for performing life analyses. The well-known Iowa type survivor curves will then be used, if possible, to estimate representative average service lives and type dispersions.

4 - To show how the expressions developed for the condition percent factors and values can be applied to a machinery and equipment account or category.

REVIEW OF PREVIOUS WORK

It was mentioned earlier that value has two basic concepts, market value and value to the owner. Differences may exist between the two measures, e.g., due to sentimentalities or for the reason that the property has peculiar value to the owner.

Whether the end result sought is market value or value to the owner, in most cases these two attributes cannot be ascertained by direct measurement. Leading writers on the subject of valuation procedures all cite the use of indirect measurement by the way of what is called "evidences of value." Bonbright puts forth as evidences of value the following (3):

1 - Actual sales

2 - "Actual cost," "original cost," or "historical cost" as the case may require

3 - Replacement cost

4 - Capitalized income.

He emphasizes the necessity of a fairly precise definition of value as a prerequisite to the intelligent discussion of the evidence. A case in point is when the use of an evidence is not permitted to be taken into consideration by law. In an ad valorem tax case before the Iowa supreme court, it was ruled that actual sales of similar machinery in the used machinery market do not constitute an evidence of the market value of the machinery in use by the company (9).

Marston et al. listed the steps they deemed necessary to establish the evidences of value and how to estimate the fair value according to these evidences (1). Their process is outlined below:

1. Determine the original cost of all fixed capital physical property, excluding land, and adjust this cost for consumed usefulness.
2. Estimate the reproduction or replacement cost of all fixed capital physical property, excluding land, and adjust for consumed usefulness of the existing property.
3. Estimate the fair market value of all land.
4. Estimate the value of the working capital.
5. Estimate the present value of the intangible property possessed by the enterprise.
6. Determine the fair rate of return for the enterprise.
7. Estimate the earning value of the enterprise.
8. Estimate the service worth (fair earning) value of the enterprise.
9. Estimate the market value on the basis of the average market prices of its outstanding securities or on the basis of the market value of similar properties.
10. Find an evidence of the total present investment in the enterprise by summing items 1, 3, 4, and 5.

11. Find an evidence of the total value based on the present cost of construction of the enterprise by summing items 2, 3, 4, and 5.

As for establishing the fair value based upon the evidences, they suggest the following:

1. Compare or contrast items 7, 8, 9, 10, and 11. (Some of these evidences of value may not be obtained for particular enterprises.)
2. Find the fair value by giving such weight as may be just and right in each case to each item and to all other factors affecting the value.

It is well to stress the fact that these authors realized that in many instances not all of the evidences may be obtained. This is a part of the problem under study here.

Although the problem of real estate appraisal is not dealt with in this study, it is worth mentioning that Babcock proposed seven methods for valuation of real estate following three general methods (8, pp. 167, 168):

1. Income method.
2. Replacement - cost method.
3. Market - comparison method.

In general, authors agree that there are three main approaches to establish evidences of value:

1. Market approach
2. Earning approach
3. Cost approach.

When market value is the desideratum and it cannot be ascertained due to the rarity or nonexistence of market exchanges of the property or comparable properties, the other two evidences of value are resorted to, namely the earning evidence and the cost evidence.

The earning or income approach uses the present worth method for discounting the expected future monetary returns to arrive at an estimate of value of the property. To illustrate how this approach works, a simple example will be given. Suppose that an investor estimates that a certain unit of depreciable property would be able to earn, after taxes and operating expenses, a uniform \$1000 per year for the next 10 years at which time he estimates that he can get \$50 for net salvage. The annual \$1000 would pay interest on the part of capital borrowed to acquire the property; plus an allowance for yearly consumption of usefulness or for depreciation of the property; plus net income on his own money invested. Now he computes what the property is worth by discounting these 10 annual returns and the net salvage with a discount rate for his specific mix of debt and equity capital commensurate with comparable opportunities, say 10%. He comes out with \$6164.275 as what the property is worth at age zero. If he acquires the property for that amount and the property earns the yearly \$1000 forecast, then the value of the machine at any other age would be the present worth of the remaining returns plus the present worth of net salvage. For example, at age 6, the value of the property would be estimated at $3204.15 by discounting, at 10%, the four remaining $1000 returns and the $50 net salvage. On the other hand if at age 6, he estimates different returns for the remaining years or different net salvage or different discount rate or that he intends to keep the property longer or to sell it earlier than the original 10 years life, he would apply the new estimations to arrive at an estimate of value at age 6. The same procedure would apply at any other age.

The annual $1000 return in the example is called "capital recovery" in Engineering Economy literature (10, 11). To aid in understanding the nature of this capital recovery, Figure 1 is presented which shows the various cash flows in an enterprise. The after-tax cash flow shown in the figure represent the capital recovery and is composed of net income plus depreciation expense plus interest on debt. It provides the owner repayment of his investment and return on it during its life.

In valuation, the same quantity is called the "operation return" and is taken to approximate the annual service derivable from the property. However, it should be pointed out that, in the valuation case, the annual depreciation component would be the annual decrease in usefulness or utility of the property which may or may not equal the annual depreciation

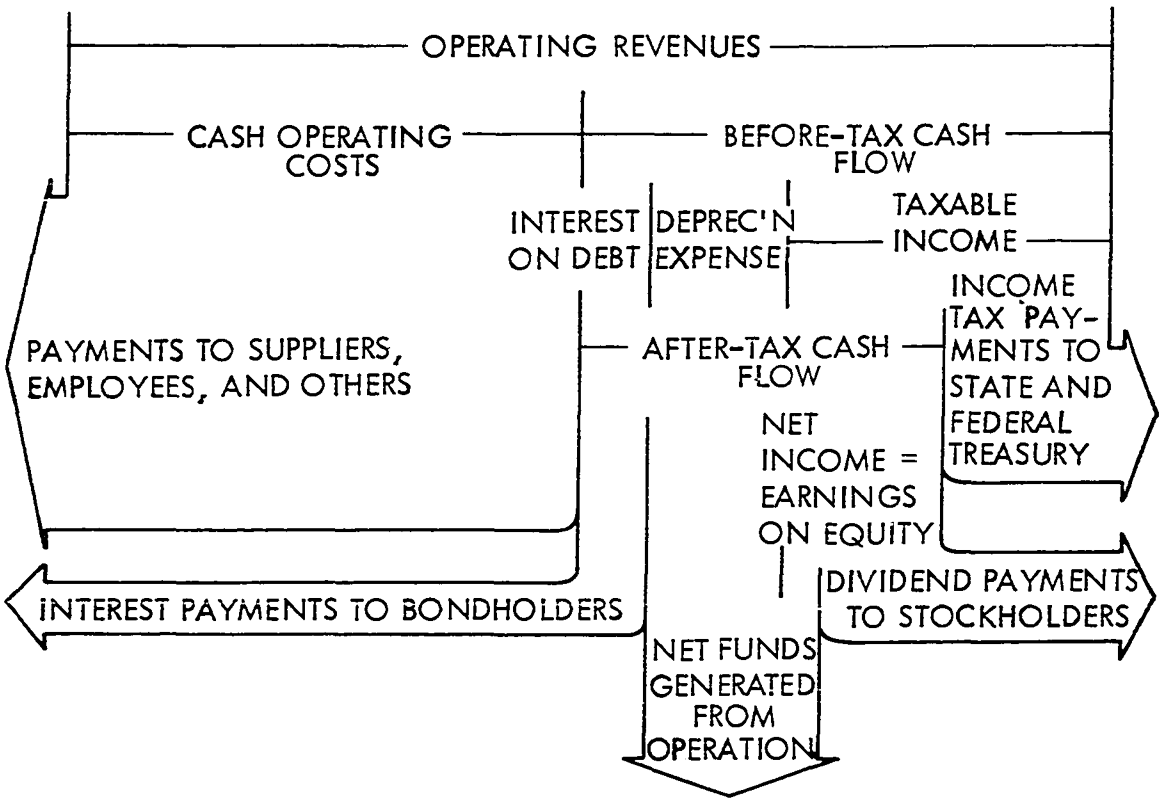

Fig. 1. Relationship of the various cash flows of accounting and economy studies (10)

expense for tax purposes shown in Figure 1. Still, the annual capital recovery will equal the annual operation return. About the operation return, Winfrey says:

> By definition, operation return includes annual depreciation and annual net return on the depreciated value including salvage value (5, p. 25).

The operation return is thought of as "quasi-rent" by another writer. Edwards states:

> When an entrepreneur contemplates the purchase of a machine he must compare the expected value of the services the machine will render to the costs he must incur in acquiring the machine. The value of the machine's services in any period is a quasi-rent and is determined (in his expectations) by deducting from the proceeds he realizes from selling the machine's output all costs incurred in producing and selling that output (raw material, labor, etc.) except depreciation on the machine and any interest costs related to the acquisition of the machine) (12, p. 48).

While the earning or income approach is applicable in the case of properties whose earnings and costs can be estimated, e.g. an apartment building, it is not possible to apply it when the earnings and costs cannot be segregated from those of the enterprise. This is exactly the problem under study and the use of the cost approach is thought to be of significant help in this case. The underlying principle of the cost approach is that the cost of the property when first acquired is strong evidence of value at that time. On this, Marston et al. state:

> When the physical property is new, its fair value is strongly evidenced by its original cost new, provided that there has been no change in potential usefulness and that the costs of original construction were prudent and normal in all respects (1, p. 347).

However, the typical valuation situation deals with property at age other than zero and the cost new evidence has to be adjusted for "depreciation." In general, the term depreciation may refer to different meanings (1, p. 175):

1. Decrease in value
2. A cost of operation
3. Physical condition.

In valuation, depreciation would mean decrease in value or the consumption of usefulness or utility of a property. Of the total decrease in value of a property at a certain age, Marston et al. states:

> The decrease in value of property from age zero to any service age results from a reduction, since it was first put into service new, in the present worth of its probable future services (1, p. 181).

At this point, the situation is that of trying to estimate the value of a property at age other than zero and whose operation returns cannot be identified by virtue of it being only a part of a larger, composite income producing entity and whose value at age zero is strongly evidenced by its cost new. The concept of value would still be the same and the value of the property at any age is estimated by the present worth of the unknown remaining services or operation returns. By reversing the calculations done for the income producing property in which value was estimated from the present worth of future operation returns, financially equivalent future operation returns over the expected life of the property can be calculated on the assumption that cost new equals value at age zero. An evidence of value at any age other than zero would then be defined as the present worth of the remaining calculated operation returns.

As might be expected the mathematics of financial equivalence are such that many varying series of operation returns can be calculated from a single cost new amount and it becomes necessary to choose a reasonable pattern. The most convenient, though not necessarily applicable in all cases, assumption is that of a series of uniform operation returns or equal annual services throughout the unit's life. About that Winfrey states:

> In a continuous property, users of the service from year to year should pay equal annual charges for net return and depreciation expense for a given item of equipment, so long as that item renders the same quantity and quality of service (5, p. 24).

However he acknowledges the possibility of variable operation returns by saying:

> The annual operation returns R will vary, but they may be represented by an equivalent uniform annual operation return R_u of such magnitude that the sum of the present worths of all R_u's receivable during the life of the unit is equal to the sum of the present worths of the actual annual operation returns R during the same life (5, p. 25).

Using the above equivalency concept, a factor called the "condition percent factor" can be calculated at any age by which the cost new or the value new evidence will be adjusted to give evidence of value at that age. Winfrey derived an expression for this condition percent factor for a unit of property which is written as (5, p. 26):

$$c = \frac{(1 + r)^n - (1 + r)^x}{(1 + r)^n - 1},$$

where

c = condition percent factor

n = probable life of unit in years

x = age of unit in years

r = rate of net return per annum.

He, also, defines it as:

> The ratio of the present value of the depreciable property relative to its depreciable value when new (5, p. 26).

When the condition percent factor is multiplied by 100, it is called the "condition percent." Tables giving the condition percent for properties at different ages for different probable lives and discount rates and using the assumption of an equivalent uniform annual operation return have been published (13). In a later publication he co-authored, the condition percent factor was called "expectancy-life factor" (1).

It is important to note here that if the actual operation returns were not uniform during the property service life, then Winfrey's assumption of the equivalent operation return, R_u, mentioned above will give some problems. For example, if the operation returns decline from year to year, the use of R_u will result in higher condition percents than what they are supposed to be using the actual declining operation returns. Moreover, the more the pattern of the operation returns deviates from the uniform pattern, the more will be the discrepancy between the computed condition percents and the actual condition percents. Marston and Agg seem to have recognized this problem and stated by introducing what they called "probable future equivalent uniform annual operation returns" and defined them as:

> . . . those estimated at any date to have the same present worth at that date as the probable future operation returns yet to be earned (14, p. 92).

Further, they introduced a ratio called the "probable future operation-return ratio (PFORR)" and gave its definition as:

> The probable future operation-return ratio (or PFORR) of any physical-property unit is the ratio of its probable future equivalent uniform annual operation returns, during its

> probable future service life, divided by its equivalent uniform annual operation returns throughout its entire service life, past and future (14, p. 92).

The authors also stated that:

> This factor should be estimated directly (not calcualted) by a qualified engineering-valuation expert, at the time he makes the examination of the unit for the purpose of forecasting its expectancy and estimating its actual depreciation (14, pp. 92, 93).

About the probable causes for the decrease in the annual operation returns they said:

> Such decrease in the annual value of the operation return earned by the unit may be due to lowered efficiency, lessened output capacity, increased maintenance costs, increased running costs, intermittent (stand-by) service, and/or operation at less than normal capacity (14, p. 92).

Later, Marston et al. introduced a similar factor called "the service factor." To understand its evolution, it is better to quote:

> Perhaps, more often than otherwise, the services of a property may be worth less in its latter periods of use than in the periods of its early service. Especially is this true of operative machinery, which reduces in operating efficiency and increases in operating costs (exclusive of depreciation cost) with age.
>
> The expectancy-life factor measures the reduction in usefulness only on a time or service unit basis, but not with regard to the quality, or value, of that service. This quality, or value, of the service may need recognition in appraisals and is in effect, when so recognized, an adjustment

> of the assumed R in the derivation to compensate for the fact that operation returns may not be uniform during the entire service life.
>
> This adjustment factor may be termed the service factor, and, as such, the service factor is one of judgment to be introduced as the appraiser may see need for its use (1, pp. 235, 236).

Although the reasons for the introduction of both the "PFORR" and the "service factor" are realistic, little use was made of them. It is suspected that the total reliance on the experience and judgment of the appraiser in setting these factors has lessened their acceptability.

The characteristic of decreasing operation returns has been investigated by other authors. Terborgh studied the matter of machinery and equipment replacement at length. In calculating the retention values of existing equipment, he proposed different patterns of decline in "absolute earnings" (operating revenue less cash operating costs), of which he says:

> These projections are alike in showing a declining trend of earnings over the estimated life, differing only in the pattern of the decline. The patterns are defined in terms of the proportion of the total lifetime runoff occurring during the first half of the life. Thus the pattern we have labeled "standard" shows a first-half runoff of one half. Variant A shows one third; and variant B, two thirds (15 , p. 71).

His absolute earnings are equal to "service values" plus income taxes. It is recognized that service values is another name for the operation returns defined earlier. He also says regarding the same point:

> There can be no dispute that for a wasting asset with a limited service life the general pattern of its earnings, as here defined, is some kind of a declining figure. They are normally highest when the asset is new; at the end of its service life they are presumably zero (else its life would be longer). Between the beginning and the end they must run off in some fashion.
>
> There is no mystery about this runoff. The asset accumulates deterioration and obsolescence as it ages. The cost of maintenance rises. Performance deteriorates. The service becomes less reliable. At the same time, improved substitutes appear, adding relative deterioration (obsolescence) to the accumulating effects of physical degeneration. All of these factors - rising operating costs, impaired service quality or adequacy, and improved alternatives - combine to reduce the competitive value of the service as the asset ages (15, p. 69).

Earlier, Terborgh studied the relation between age and intensity of use for eight classes of productive equipment: Locomotives, farm implements, truck-tractors, trailers, tractors, trucks, intercity buses and passenger cars. Of the results of this study he says:

> While these eight types of equipment are not so broadly representative as one could wish, overweighted as they are with motor vehicles, they present a consistent pattern of service intensity

> declining with age that is undoubtedly characteristic of standard productive equipment generally (16, p. 19).

The curves showing the relation between age and intensity of use for those eight types of equipment are reproduced in Figure 2 for reference. About the interpretation of the curves, he cautions:

> It should be pointed out that these measures of declining use intensity compare units now old with other similar units now new, not old units with their own past. Mileage run or hours worked per year during the youth of units now old undoubtedly differed considerably from the current figures for presently young units. The results must be taken therefore as only roughly indicative of the course of use over the lives of identical units (16, pp. 20-21).

Terborgh also studied the relation between repair costs and age for eight classes of equipment: metal-working machinery, textile machinery, light trucks, intercity buses, locomotives, farm implements, passenger automobiles, and local buses. Figure 3 shows the resulting curves of his study about which he comments:

> These trend curves, which presumably reflect fairly closely what the average would show for a large and representative sample of each type of equipment, display a general tendency for repair costs per unit of service to rise somewhat more rapidly in the early stages of service than later. It is interesting to note, however, that whether these costs are related to age or to accumulated use, the curvature appears very mild (16, p. 69).

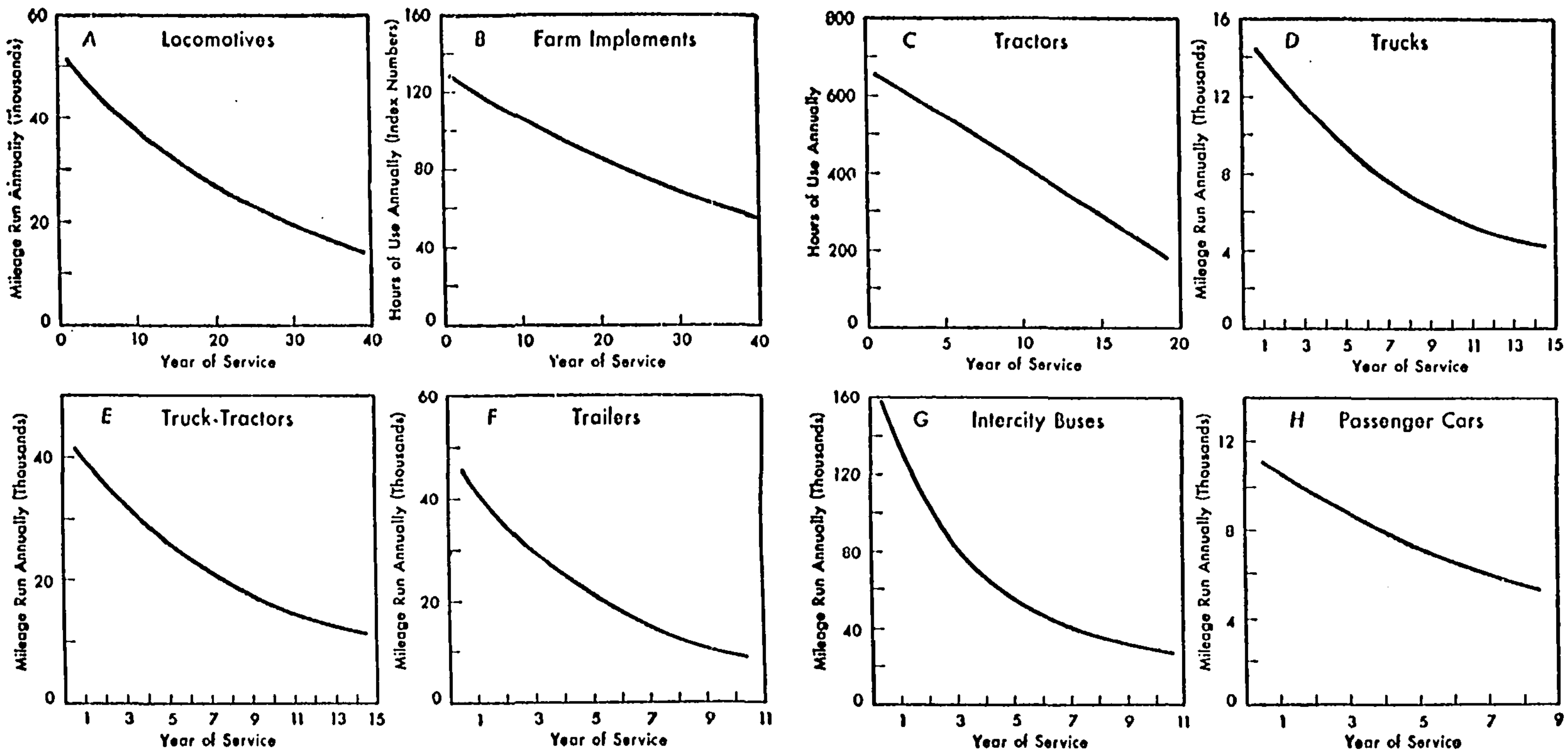

Fig. 2. Relation between age and intensity of use of eight classes of equipment (16)

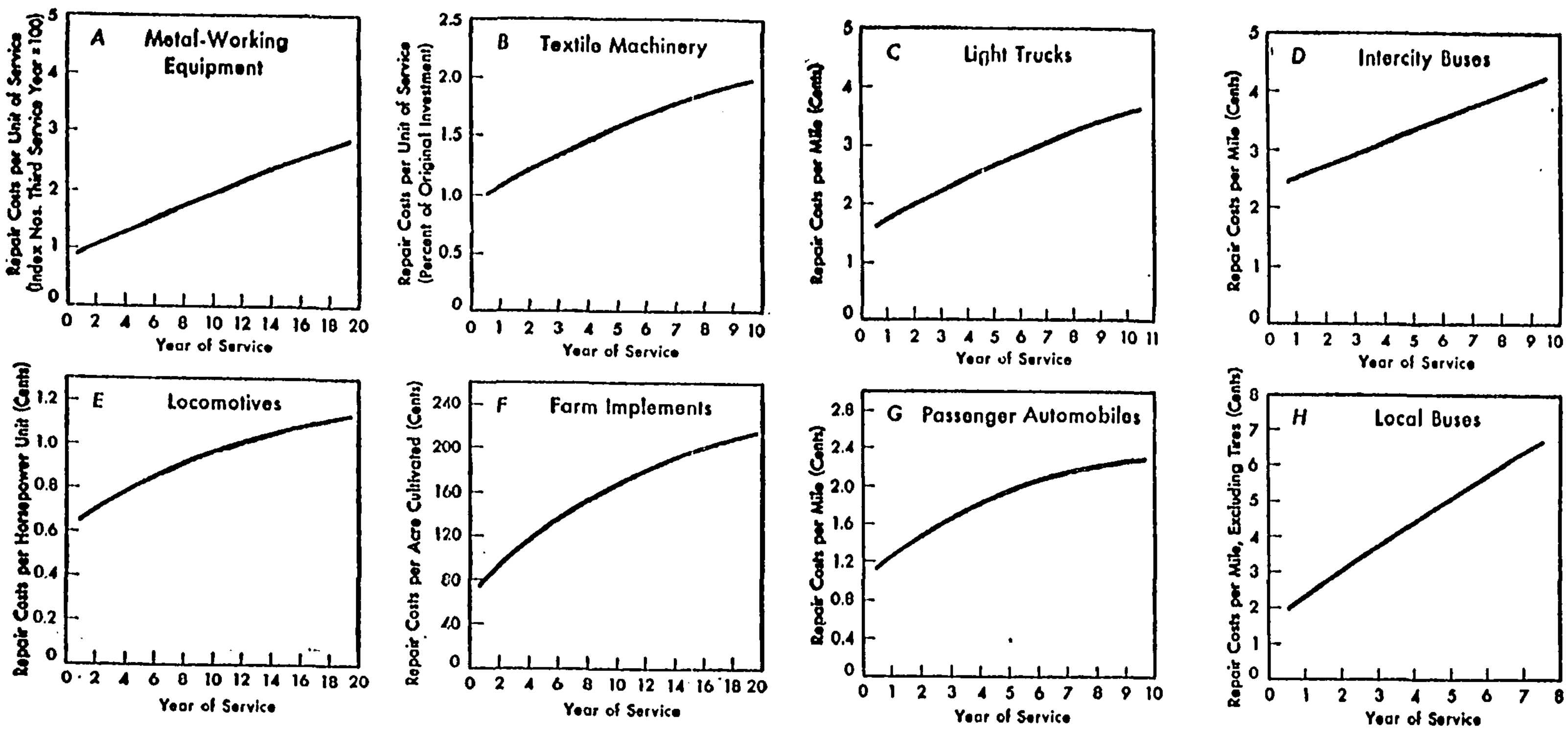

Fig. 3. Relation between age and repair cost per unit of service for eight classes of equipment (16)

Another study related to the intensity of use of property was done by Egg (17). This study included the experience with 387 fossil fuel-fired steam power production units in fifteen large electric utilities. The results showed that as the production units age, their annual KWHR declines. This was interpreted to be due to the system growth with the consequence of older units becoming obsolete and relegated from base load to peak load status. Figure 4 shows the change of the annual output with age for a composite of twelve companies out of the fifteen companies studied (18). It shows a definite declining pattern of annual output with age of power production units. This situation can be seen as a decline of "annual service" of those production units with age.

While other units in other industries may not experience the fast decline in annual services as in the case of the power production units, some would do due to pronounced effects of obsolesence and the use of the units in secondary uses due to economy of operation, system growth or any other reason.

Concerning the maintenance cost, Lipas states that according to Niini there are three practical possibilities regarding the maintenance cost (19, pp. 89, 90):

1 - The class of service is allowed to fall and the intensity of operation to diminish; maintenance cost is kept constant.

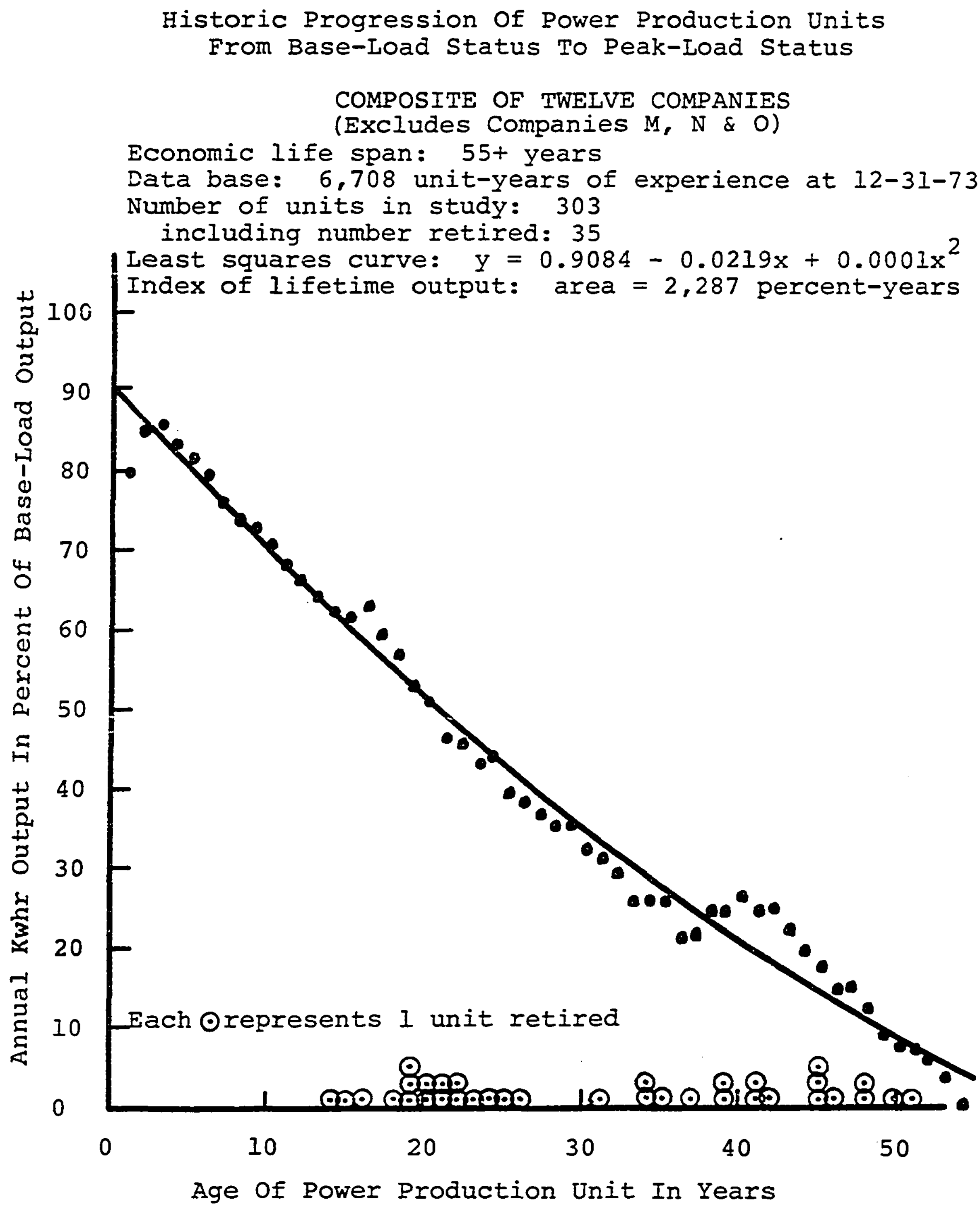

Fig. 4. Change of annual output of power production units with age for twelve companies (18)

2 - An attempt is made to keep the class of service unchanged; constant intensity of operation. Maintenance cost rises linearly.

3 - The class of service is made remain [sic] unchanged at first but is later allowed to fall gradually. Maintenance cost rises at first but levels off gradually.

Babcock proposes four types of future income streams for buildings. He defines those income premises as:

> Premise 1 assumes the future income to occur in equal annual installments of $1 each commencing one year from the present time and running for n years.
>
> Premise 2 assumes the future income to occur in annual installments commencing with an installment of $1 one year from the present time and running for n years, the successive income installments declining to zero in the n + 1 year and being proportional to the annual amounts found in the value curve corresponding to a level annuity at 10 percent for n years.
>
> Premise 3 assumes the future income to occur in annual installments commencing with an installment of 1 $ one year from the present time and running for n years, the successive income installments declining to zero in the n + 1 year and being proportional to the annual amounts found in the value curve corresponding to a level annual annuity at 3-1/2 percent for n years.
>
> Premise 4 assumes the future income to occur in annual installments commencing with an installment of $1 one year from the present time and running for n years, the successive income installments declining to zero in the n + 1 year by equal differences, that is, as a straight line (8, pp. 412-414).

In real life situations a unit of property or a human being is a part of a parent population. The statistical method of arranging and studying human births and deaths has been in use a long time by insurance actuaries to determine human life expectancy and corresponding insurance premium rates. Survivor curves, showing the number of persons who survive the various ages of life, have been used for determining those insurance rates for some 200 years (1, p. 142). However, the first reported case of compiling mortality tables for physical property dates only back to 1903 (20, p. 10).

For economics of bookkeeping, accounts in industrial enterprises are usually held for a number of units installed at the same time and called a vintage group or held for several vintage groups. Therefore the analyses of the life characteristics of groups of property are of major practical importance. When several units are placed in service at the same time, chances are that those units will not be retired from service at the same time. Some units will be retired before others. This phenomenon is called life dispersion.

While both life analysis of past mortality experience and life forecast of remaining service of the property are needed, study of life characteristics of the property unit or group of units starts with life analysis. The basic idea of life analysis is to determine from historical records the dispersion of lives actually experienced in the past by the subject population (21). This dispersion is represented by a retirement frequency curve (Figure 5) which relates the number or percentage of retirements from some original placement to the property's age. The actual or observed frequency curve is often erratic and hard to analyze and consequently its cumulative form, the survivor curve shown on the same figure is more commonly utilized. The survivor curve indicates the percentage of an original placement of property that remains in service at ages zero to maximum life.

The retirement frequency and survivor curves are related by the following expressions (21): If Y_j represent the decimal portion retired from an original placement in the j^{th} age interval, then

$$\sum_{j=1}^{n} Y_j = 1.0$$

where n is the age interval in which the last survivor of the placement is retired. The cumulative retirement fraction, R_X, from an original placement over the time span from age zero to the end of the X^{th} age interval would be

$$R_X = \begin{cases} \sum_{j=1}^{X} Y_j & X = 1,2,3,4, \ldots, n \\ 0.0 & \text{if } X = 0 \\ 1.0 & \text{if } X = n \end{cases}$$

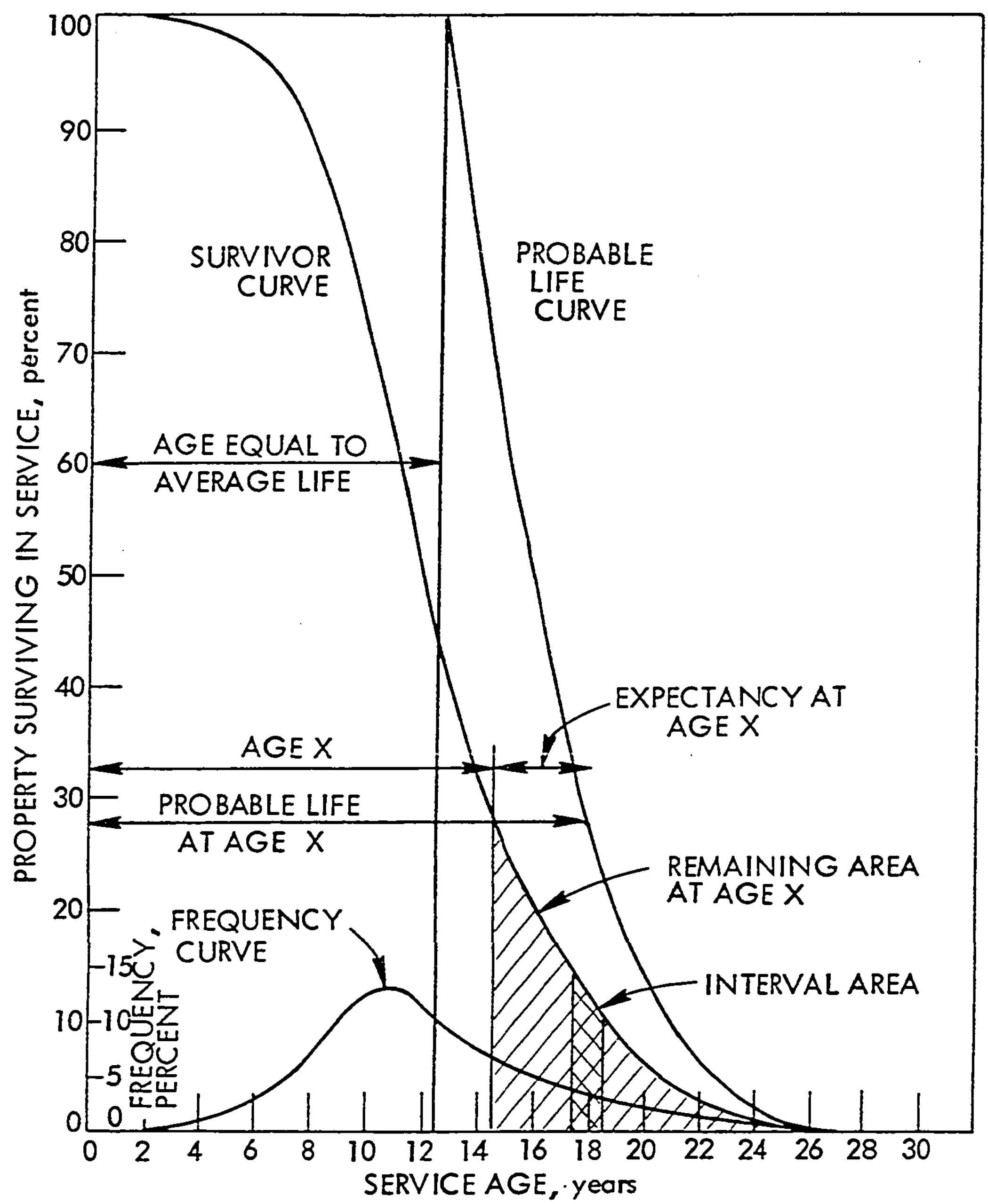

Fig. 5. The survivor curve, frequency curve, and probable life curve and their nomenclature (1)

Correspondingly, the portion of an original placement remaining in service at the end of the X^{th} age interval would be

$$S_X = 1 - R_X \qquad X = 0,1,2,3,\ldots, n$$

S_X is the discrete version of the survivor curve.

The retirement dispersion of a group of units can also be represented in terms of a series of retirement ratios for successive age intervals. A retirement ratio for an age interval is the amount of property retired during the age interval divided by the amount of property surviving at the beginning of the age interval and exposed to retirement, or

$$r_j = \frac{Y_j}{S_{j-1}}$$

and a survivor ratio for an age interval will be defined as

$$s_j = 1 - r_j$$

The retirement ratio and survivor ratios are related to the survivor curve expression as

$$S_X = S_O \prod_{j=1}^{X} (1 - r_j) = S_O \prod_{j=1}^{X} s_j$$

These relationships can also be shown in terms of continuous functions (22).

The retirement frequency function is the probability function of the service life and the retirement ratio during any period is a conditional probability of retirement during that period (22). Also, the survivor curve represents the probability that a property will survive to a certain age. For example, the probability that a property will live to age (X + 1) equals:

probability a property will live to age 1

times probability a property will live to age 2 given it has lived to age 1

times probability a property will live to age 3 given it has lived to age 2

times --------------

times probability a property will live to age X given it has lived to age (X - 1)

times probability a property will live to age (X + 1) given it has lived to age X.

Primarily, the observed life dispersion pattern is used to measure life realized or to provide a basis for predicting remaining life. If the observed retirement frequency curve is complete, a weighted average or mean life can be computed. This mean life is called the average service life. If the observed retirement frequency curve is believed to be a good and reasonable prediction of how the retirements will occur in the future from a new property group at age zero, the mean life calculated is termed the probable average service life or the forecasted life of the average or typical property unit at age zero. The mean life of the retirement frequency curve is given by (25, p. 8):

$$\frac{af_a + bf_b + cf_c + \ldots + nf_n}{f_a + f_b + f_c + \ldots + f_n}$$

where f_r is the frequency corresponding to the value r of the variable (age in this case). The mean life could also be described as the position of the ordinate through the center of gravity of the retirement frequency distribution.

As mentioned before, the retirement frequency curve is hard to analyze and the survivor curve is usually used in life analysis. The same mean life computed from the retirement frequency curve can be computed from the survivor curve. In this case it is equal to the area under the survivor curve divided by the original placement at age zero or equal to the expected service life at age zero (22, p. 61). Life expectancy at any age is that period of time from the present age to the age when the unit will probably be retired from service. An estimate of expectancy for a typical unit at age X is determined by finding the remaining area under the forecasted survivor curve to the right of that age and dividing the amount by the percent surviving at age X. The probable life of an item or group of items of the same age is, by definition, equal to the age of the property plus the expectancy estimated as of that age. Figure 5 shows the survivor curve, frequency curve, and probable life curve and their nomenclature (1,

p. 147).

In the majority of cases, the retirement experience of groups of property produces incomplete survivor and frequency curves because at the time of the study there would be still some surviving units. These incomplete curves are called stub curves. Moreover, the observed curves are often erratic. About this Winfrey comments:

> Because of small numbers of units or infrequent and irregular observations, or both, the resulting curves possess irregularities which are not natural to the general behavior of large properties (23, p. 37).

The stub, and most often erratic, survivor curve must be smoothed and extended to zero percent surviving before the probable average service life can be computed. Extension and smoothing of the observed survivor curves can be accomplished by judgment, statistical curve fitting, or by matching to standard or type curves (1, pp. 164, 165). The judgment method involves extending and smoothing the curve by eye along the most probable path and depends greatly on the experience of the analyst. Statistical methods involve fitting equations to the data and include the Gompertz-Makeham, Weibull, and polynomial methods (24). Matching to standard or type curves involves use of a previously established set of standard or type curves which are thought to be representative in shape to those likely to be encountered in practice. Some of these are the Iowa Type Curves, Gompertz-Makeham Curves, Patterson curves, and New-York h-curves (22). The Iowa Type Curves will be utilized for the purposes of this study and a brief summary of their evolution would be appropriately given here.

Kurtz pioneered by developing the first Type Curves for industrial physical property. He studied the life characteristics of 52 groups of physical property and found that their frequency curves could be grouped into 7 types (20). The frequency curves of these 7 types were then found to fit well the Pearson frequency curves of Type I (20, p. 95). A complete discussion of Pearson frequency curves is given by Elderton (25). Later, Winfrey and Kurtz expanded the original 52 groups of property studied into 65 groups (26). They were then able to group them into 13 groups or types. About the classification process they say:

> A frequency curve on the percent of average life basis was then drawn for each of the 65 properties and the curves compared. The comparison brought out three distinguishing characteristics: location of the mode (the point of maximum ordinate) relative to the average life, magnitude of the mode, and the maximum age in percent of the average life. The curves were then classified into three groups according to whether the mode was to the left, approximately coincident, or to the right of the average life, and these three groups sub-classified in accordance with the magnitude of the mode. The classification, which was almost wholly by inspection, resulted in 13 groups or types, four groups having the mode to the left of the average life, five groups having the mode at the average life, and four groups having the mode to the right of the average life (26).

Winfrey studied an additional 111 property groups (23). This study confirmed the results obtained from the earlier 65 groups and resulted in the identification of 5 new types, all of which are extensions of the 3 original families of curves.

Couch added 4 type curves (27). These curves are called origin-modal from the fact that the mode is at or very near the origin. One of them is the straight line survivor curve and the other three are J-shaped or exponential type survivor curves. In total, the Iowa Type Curves now number 22, i.e., seven symmetrical, five right-modal, six left-modal, and four origin-modal (23). Figures 6 to 9 show the final survivor, probable-life, and frequency curves for the 4 families of the Iowa Type Curves (23).

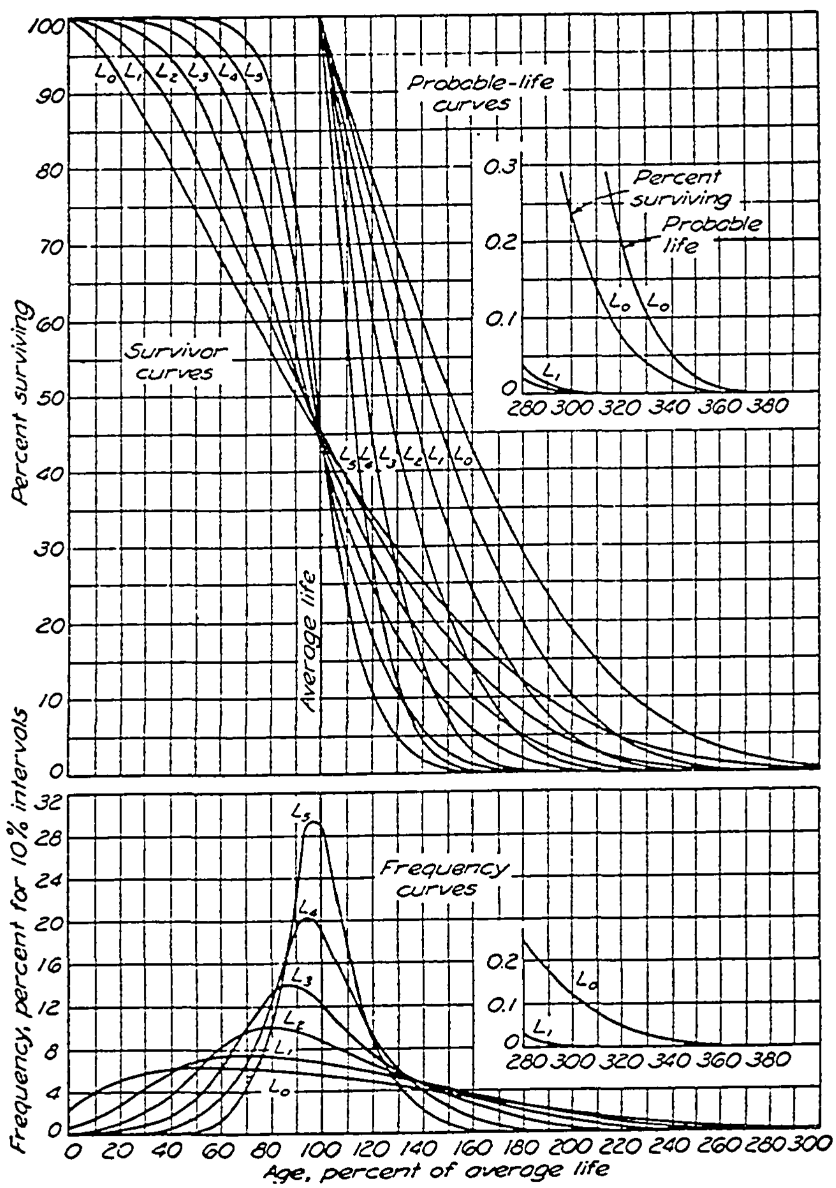

Fig. 6. Final survivor, probable life, and frequency curves for the left-modal types (23)

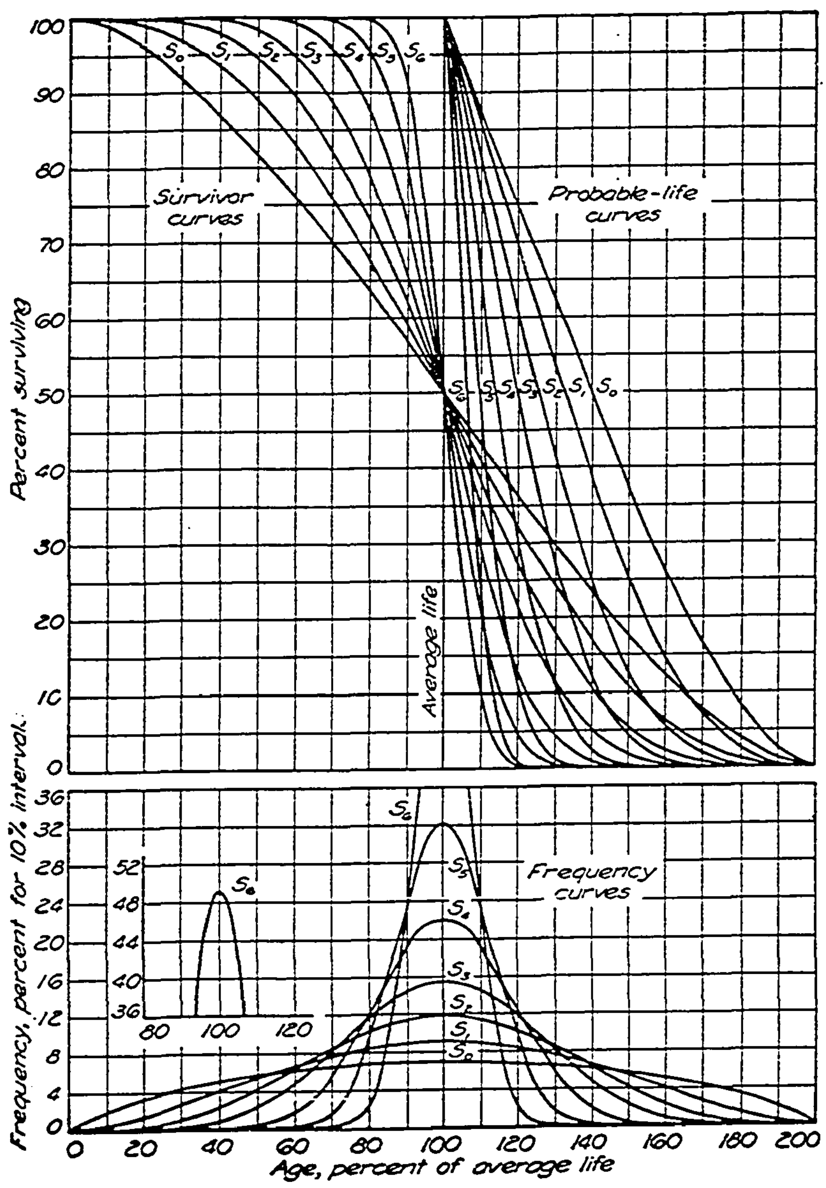

Fig. 7. Final survivor, probable life, and frequency curves for the symmetrical types (23)

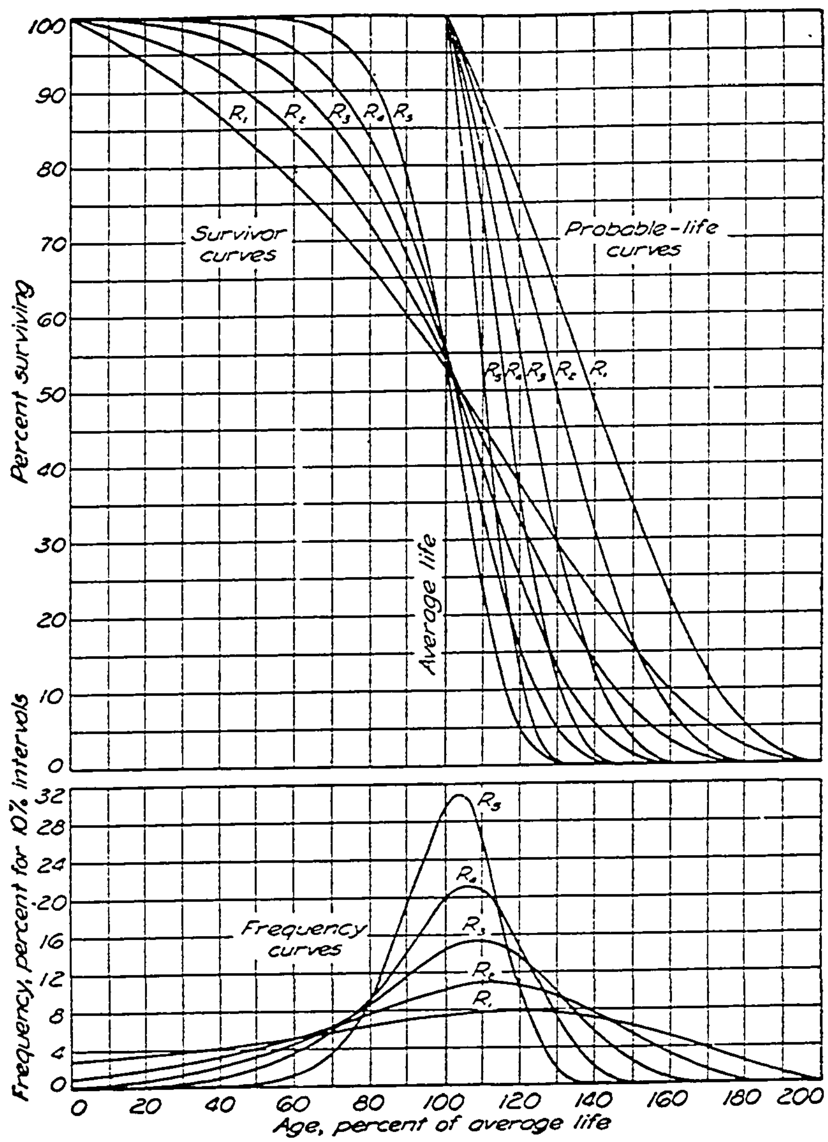

Fig. 8. Final survivor, probable life, and frequency curves for the right-modal types (23)

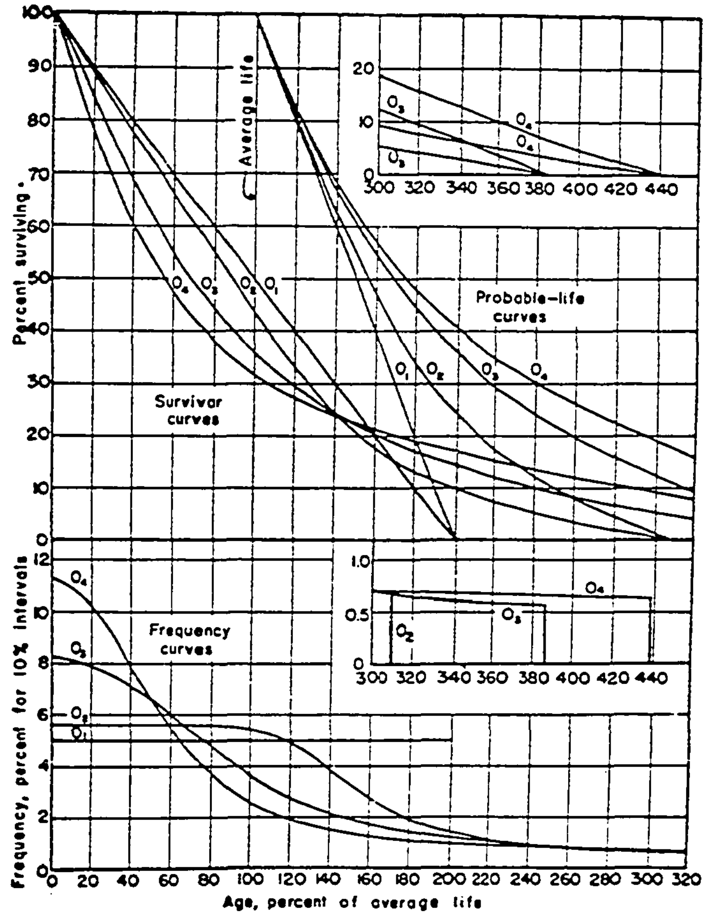

Fig. 9. Final survivor, probable life and frequency curves for the origin-modal types (23)

MODEL DEVELOPMENT

The variation with time of the service worths of industrial property is of paramount importance in ascertaining evidences of values at different ages and especially at ages other than zero. At age zero, the value of the equipment is strongly evidenced by its cost new. It is desired to establish evidences of value of machinery and equipment at ages other than zero and whose annual earnings and operating costs cannot be segregated from the earnings and operating costs of the whole enterprise. In this context, the annual operation returns, composed of annual recovery of the investment plus the annual return on the unrecovered investment, are considered the measures of the annual service worths of the machinery and equipment in question. It is suspected that in many situations the service worths of the machinery and equipment decline by the passage of time. Whether the property continues to perform its original assignment or is degraded to lower job or jobs during its life, the decline in its annual service worth can occur. The former case can be illustrated by a machine producing the same quantity of the product while its annual operating costs are rising, or, a machine producing increasing percentage of rejects as it ages, or a machine which is superseded on the market by better and more economical substitute which relatively increases its cost of operation.

The latter case of job degradation can take many forms. The machine can be assigned the production of special orders with intermittent operation, or it can be put into stand-by operation, or it can be used for peak load generation as, e.g., in the electrical power industry. In these cases obsolescence is thought to be the main factor of degradation (16, p. 72). Obsolescence is defined in many ways and the one most repeated in the literature is that obsolescence is functional depreciation (1, p. 140; 2, pp. 185, 186). However, Bonbright disagrees with this definition and uses the general principle of relative utility in defining obsolescence. A quote is in order:

> By invoking the notion of relative utility, one may extend depreciation to cover obsolescence. The old reciprocating steam engine may be just as efficient as it ever was; but as compared to a modern turbine it may have become relatively inefficient (3, p. 183).

Obsolescence can be envisioned as accumulating at a constant rate on the average (16, p. 68) or occurs in discrete steps (28). Although this discussion would be helpful if obsolescence can be separated and measured, the fact remains, for the purpose of this inquiry, that due to the forces of obsolescence and other operating and economic factors, the service worths of industrial machinery and equipment may decline with age. The expressions developed hereafter are expressly designed to allow this phenomenon to be taken into consideration. Before undertaking the model development, it might be beneficial to illustrate by a simple example the possible erroneous results which could occur if the variable annual operation returns are represented by an equivalent uniform annual operation return (5, p. 25).

Suppose that a property is forecasted to last for 3 years and to have operation returns of \$1000, \$900 and \$800 for the three consecutive years and zero net salvage. At a 6% effective annual discount rate, its present value would be:

$$P_o = 1000(1.06)^{-1} + 900(1.06)^{-2} + 800(1.06)^{-3}$$

$$= \$2416.10$$

The computed equivalent uniform annual operation return, R_u, can then be calculated as:

$$R_u = P_o \left[\frac{i\ (1 + i)^N}{(1 + i)^N - 1}\right] = 2416.10 \left[\frac{0.06(1.06)^3}{(1.06)^3 - 1}\right]$$

$$= \$903.90$$

Now a comparison will be made between results obtained for the present value and the condition percent at age 1 and calculated using both the actual operation returns and the computed uniform "equivalents." At age 1, the present value P_1' using the actual operation returns is \$1561.06 contrasted with the present value P_1 of \$1656.85 using the "equivalents."

Consequently, the condition percent $C_1'(= 100 \frac{P_1'}{P_o})$ at age 1 using the actual operation returns is 64.61% while the condition percent $C_1(= 100 \frac{P_1}{P_o})$ using the equivalent R_u would be 68.58%. Differences will arise at age 2 as well. In general, these differences will be greater, the higher the rate of discount. Also, differences will result in the opposite direction if the actual operation returns are increasing instead of decreasing.

The moral of the above discussion is that, for variable service worths or operation returns, more realistic and representative expression for the operation returns is needed to arrive at reasonable results for the condition percent and the value of property.

In many instances, it is desired to know the condition percent and the value of a property unit or group of units at half-year ages, e.g. at ages 1/2, 1-1/2, 2-1/2, ... etc. years instead of or in addition to whole-year ages of 1, 2, 3, ... etc. years. To serve this requirement and to represent more closely the usually nonuniform cash flow, the operation returns will be assumed to be semi-annual and will be treated as lump-sum end-of-period amounts. With this in mind, the following symbols will be used in the derivation of the model equations:

N = probable life of property unit or frequency group in half-year periods

X = age of unit or property group in half-year periods

V_N = value new of unit or of survivors of a property group

V_X = value at age X of a unit or of survivors of a property group

V_S = estimated net salvage value

S = salvage ratio = (V_S/V_N)

V_{ND} = depreciable value = $V_N - V_S = V_N(1 - S)$

R_X = operation return for age interval $(X - 1)$ to X and treated as an end-of-period quantity

r = effective annual discount rate

i = effective semi-annual discount rate = $(1 + r)^{0.5} - 1$

$q = (1 + i)$

T = progression rate of operation returns

$(p/f)_N^i$ = present worth of a future sum

$= (1 + i)^{-N}$

$(p/a)_N^i$ = present worth of a uniform series

$$= \left[\frac{(1 + i)^N - 1}{i(1 + i)^N}\right]$$

$(a/p)_N^i$ = uniform series worth of a present sum or capital recovery factor

$$= 1/(p/a)_N^i$$

$(p/g)_N^i$ = present worth of a gradient series

$$= \frac{1}{i} \left[(p/a)_N^i - N(p/f)_N^i \right] .$$

As discussed earlier, the operation returns of a property unit through its service life could be uniform or variable from period to period. It was also shown that the calculation of an equivalent operation return from an actual variable operation return pattern produced incorrect results for the condition percent and the value of property at any age other than zero. As for the variability of the operation returns, literature cited and data reported later support the notion of declining operation returns with age for industrial machinery and equipment in general.

Accordingly, it was natural to try to suggest a valuation model which offer the possibility of representing the case of declining operation returns with age of property. In this context, the following expressions for the operation returns were investigated:

1. $R_X = R_1 \cos \left[(X - 1) \left(\frac{90}{N} \right) \right]$

   A similar thought of using a trigonometric function was suggested by Benjamin where he proposed the use of a depreciation method based on a sine curve to write off assets (29, p. 303).

2. $R_X = R_1(1 + T)^{X-1}$

This model is used in engineering economy problems involving geometric progression (10, p. 50).

3. $$R_X = R_1 \left[\frac{T^N - T^{X-1}}{T^N - 1}\right] \qquad (1)$$

Model Proposed

For developing a general model, the third expression above is chosen because of its application flexibility. It will be shown that, among infinite modes, it represents the case of uniform periodic operation returns (when $T = \infty$) and also the case of straight line decline of operation returns with age of property (when $T = 1$).

In this expression, $R_{N+1} = 0$ and the last period the property would have an operation return is N. Also, the progression rate T can be any value greater than zero.

According to the present worth principle, the value of a property is the present worth of its future operation returns. The value new, V_N, can then be written as:

$$V_N = R_1 (p/f)_1^i + R_2 (p/f)_2^i + \ldots + R_N (p/f)_N^i + V_S (p/f)_N^i \qquad (2)$$

$$= R_1 (p/f)_1^i + R_1 \left[\frac{T^N - T}{T^N - 1}\right] (p/f)_2^i + \ldots + R_1 \left[\frac{T^N - T^{N-1}}{T^N - 1}\right] (p/f)_N^i$$

$$+ V_S (p/f)_N^i \qquad (3)$$

$$= \frac{R_1}{T^N - 1} \left[\sum_{M=1}^{N} (T^N - T^{M-1}) (p/f)_M^i\right] + V_S (p/f)_N^i \qquad (4)$$

From which R_1 can be expressed as:

$$R_1 = \frac{V_N [1 - S (p/f)_N^i] [T^N - 1]}{\sum_{M=1}^{N} [T^N - T^{M-1}] (p/f)_M^i} \qquad (5)$$

The ratio (R_X/V_N) can be called the operation return ratio and is expressed as follows using Equations 1 and 5:

$$\frac{R_X}{V_N} = \left[\frac{[1 - S (p/f)_N^i] [T^N - T^{X-1}]}{\sum_{M=1}^{N} T^N (p/f)_M^i - T^{-1} \sum_{M=1}^{N} T^M (p/f)_M^i}\right] \qquad (6)$$

Now applying the present worth principle at any other age X, V_X will be:

$$V_X = R_{X+1}(p/f)^i_1 + R_{X+2}(p/f)^i_2$$

$$+ \ldots + R_N(p/f)^i_{N-X} + V_S(p/f)^i_{N-X} \quad (7)$$

Substituting the values of R_{X+1}, R_{X+2}, ... etc. using Equation 1, V_X becomes:

$$V_X = R_1\left[\frac{T^N - T^X}{T^N - 1}(p/f)^i_1 + \frac{T^N - T^{X+1}}{T^N - 1}(p/f)^i_2\right.$$

$$\left. + \ldots + \frac{T^N - T^{N-1}}{T^N - 1}(p/f)^i_{N-X}\right] + V_S(p/f)^i_{N-X} \quad (8)$$

$$= \frac{R_1}{T^N - 1}\left[\sum_{M=1}^{N-X}\{T^N - T^{M+X-1}\}(p/f)^i_M\right] + V_S(p/f)^i_{N-X} \quad (9)$$

Substituting the value of R_1 from Equation 5 into Equation 9, the expression for V_X will be:

$$V_X = \left[[V_N - V_S(p/f)^i_N]\frac{\sum_{M=1}^{N-X}\{T^N - T^{(M+X-1)}\}(p/f)^i_M}{\sum_{M=1}^{N}\{T^N - T^{(M-1)}\}(p/f)^i_M}\right] + V_S(p/f)^i_{N-X} \quad (10)$$

Multiplying both sums by T and rearranging Equation 10, V_X can be written as:

$$V_X = \left[[V_N - V_S(p/f)^i_N] \frac{T^{N+1} \sum_{M=1}^{N-X} (p/f)^i_M - T^X \sum_{M=1}^{N-X} T^M (p/f)^i_M}{T^{N+1} \sum_{M=1}^{N} (p/f)^i_M - \sum_{M=1}^{N} T^M (p/f)^i_M} \right]$$

$$+ V_S(p/f)^i_{N-X}$$

$$= [V_N - V_S(p/f)^i_N] C'_X + V_S(p/f)^i_{N-X}$$

$$= V_N \left[C'_X(1-S) + S[\{1-(p/f)^i_N\} C'_X + (p/f)^i_{N-X}] \right] \qquad (11)$$

$$= V_{ND} C'_X + V_S[\{1 - (p/f)^i_N\} C'_X + (p/f)^i_{N-X}] \qquad (12)$$

C'_X will be called the "modified condition percent factor" to differentiate it from the condition percent factor derived by Winfrey (5) for the specific case of uniform annual operation returns. C'_X is expressed as:

$$C'_X = \frac{T^{N+1} \sum_{M=1}^{N-X} (p/f)^i_M - T^X \sum_{M=1}^{N-X} T^M (p/f)^i_M}{T^{N+1} \sum_{M=1}^{N} (p/f)^i_M - \sum_{M=1}^{N} T^M (p/f)^i_M} \qquad (13)$$

For long property lives, Equation 13 for C_X' will be tedious for hand calculation and closed forms for the sums are preferred. In the following, the four sums in Equation 13 will be converted into closed forms and consequently C_X'.

$$\sum_{M=1}^{N} (p/f)_M^i = (1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3} + \ldots + (1+i)^{-N} \quad (14)$$

This is the same as the interest factor for a uniform series and can be written as:

$$\sum_{M=1}^{N} (p/f)_M^i = (p/a)_N^i = \frac{(1+i)^N - 1}{i(1+i)^N} \quad (15)$$

Similarly, $\sum_{M=1}^{N-X} (p/f)_M^i$ will be:

$$\sum_{M=1}^{N-X} (p/f)_M^i = (p/a)_{N-X}^i = \frac{(1+i)^{N-X} - 1}{i(1+i)^{N-X}} \quad (16)$$

The same results can be obtained by using the zeta transform of discrete time series. Hill and Buck suggested its use in the analysis of economic problems (30). Their derivation for a uniform series, starting h periods from now and lasting (K-1) periods is given by:

$$p(i) = \frac{1}{i} [(1+i)^{1-h} - (1+i)^{1-k}] \quad (17)$$

For our first sum, h = 1 and (k-1) = N. Substituting these quantities into Equation 17, p(i) will be:

$$p(i) = \frac{1}{i}\,[1 - (1+i)^{-N}] = \frac{(1+i)^{N} - 1}{i(1+i)^{N}} \qquad (18)$$

which is the same expression arrived at in Equation 15. Similarly, Equation 16 can be proved by the same procedure.

Regarding the other sums in Equation 13, the following standard expression can be used (31, p. 1):

$$\sum_{K=1}^{N} a\,Q^{K-1} = \frac{a(Q^{N} - 1)}{Q - 1} \qquad (19)$$

Eliminating a and multiplying both sides by Q, Equation 19 can be written as:

$$\sum_{K=1}^{N} Q^{K} = Q\left[\frac{Q^{N} - 1}{Q - 1}\right] \qquad (20)$$

Applying Equation 20 to the other sums in Equation 13, the following equations will result:

$$\sum_{M=1}^{N} T^{M}\,(p/f)_{M}^{i} = \sum_{M=1}^{N} \left(\frac{T}{1+i}\right)^{M}$$

$$= T(1+i)^{-1}\left[\frac{T^{N}\,(1+i)^{-N} - 1}{T\,(1+i)^{-1} - 1}\right] \qquad (21)$$

Similarly;

$$\sum_{M=1}^{N-X} T^{M}(p/f)_{M}^{i} = T(1 + i)^{-1} \left[\frac{T^{N-X}(1+i)^{X-N} - 1}{T(1+i)^{-1} - 1}\right] \tag{22}$$

Substituting (1 + i) by q and substituting Equations 15, 16, 21 and 22 into Equation 13, C_X' can be expressed as:

$$C_X' = \frac{T^{N+1}\left[\frac{q^{N-X}-1}{i\ q^{N-X}}\right] - T^{X}\ T\ q^{-1}\left[\frac{T^{N-X}\ q^{X-N} - 1}{T\ q^{-1} - 1}\right]}{T^{N+1}\left[\frac{q^{N} - 1}{i\ q^{N}}\right] - T\ q^{-1}\left[\frac{T^{N}\ q^{-N} - 1}{T\ q^{-1} - 1}\right]} \tag{23}$$

Dividing the numerator and denominator of Equation 23 by $q^{-1}\ T^{N+1}$, C_X' will be:

$$C_X' = \frac{\left[\frac{q^{N-X} - 1}{i\ q^{N-X-1}}\right] - \left[\frac{q^{X-N} - T^{X-N}}{T\ q^{-1} - 1}\right]}{\left[\frac{q^{N} - 1}{i\ q^{N-1}}\right] - \left[\frac{q^{-N} - T^{-N}}{T\ q^{-1} - 1}\right]} \tag{24}$$

Multiplying the numerator and denominator of Equation 24 by $(i\ q^{N-X-1})$, C_X' becomes:

$$C_X' = \frac{q^{N-X} - 1 - i\left[\frac{q^{-1} - q^{N-X-1}\ T^{X-N}}{T\ q^{-1} - 1}\right]}{q^{N-X} - q^{-X} - i\left[\frac{q^{-X-1} - q^{N-X-1}\ T^{-N}}{T\ q^{-1} - 1}\right]} \tag{25}$$

Multiplying again by $(Tq^{-1} - 1)$, Equation 25 becomes:

$$C'_X = \frac{Tq^{-1}\,q^{N-X} - q^{N-X} - Tq^{-1} + 1 - iq^{-1} + iq^{N-X-1}\,T^{X-N}}{Tq^{N-X-1} - q^{N-X} - Tq^{-X-1} + q^{-X} - iq^{-X-1} + iq^{N-X-1}\,T^{-N}}$$

$$= \frac{q^{N-X-1}(T + iT^{X-N}) - q^{-1}(T+i) - q^{N-X} + 1}{q^{N-X-1}(T+iT^{-N}) - q^{-X-1}(T+i) - q^{N-X} + q^{-X}} \qquad (26)$$

This is the general expression for the modified condition percent factor C'_X in closed form.

Special Cases

There are special cases which may encounter the analyst and it is handy to have the general equations for the operation return ratio, the modified condition percent and the value already reduced to the proper forms. Those cases are:

- The case when $T = 1$, i.e. the operation returns decline by equal amounts every age period and equal zero by the end of the period $(N + 1)$.
- The case when $T = \infty$. This will be shown to represent the case when the operation returns are constant from period to period through the life of property.
- The case when $i = 0$.
- The case when $T = 1$ and $i = 0$.
- The case when $T = \infty$ and $i = 0$.

In the following sections, the equations for each special case will be developed.

The case when T = 1

Substituting T by 1 in Equation 26, C_X' will be:

$$C_X' = \frac{q^{N-X-1}(1+i) - q^{-1}(1+i) - q^{N-X} + 1}{q^{N-X-1}(1+i) - q^{-X-1}(1+i) - q^{N-X} + q^{-X}} \tag{27}$$

Recognizing that (1+i) = q, Equation 27 will result in an indeterminate form of (0/0). Applying L'Hospital's rule on Equation 26 by differentiating the numerator and denominator with respect to T and calculating the limit, C_X' can then be expressed as:

$$C_X' = \lim_{T\to 1} \frac{q^{N-X-1}[1+i(X-N)T^{X-N-1}]-q^{-1}}{q^{N-X-1}[1+i(-N)T^{-N-1}]-q^{-X-1}} \tag{28}$$

$$= \frac{q^{N-X}[1 + iX - iN] - 1}{q^{N-X}[1 - iN] - q^{-X}} \tag{29}$$

To derive an expression for the operation return ratio when T = 1, R_1 will be derived first from Equation 5 as follows:

$$R_1 = \lim_{T \to 1} \frac{V_N[1 - S(p/f)^i_N]\; NT^{N-1}}{N \sum_{M=1}^{N} T^{N-1}(p/f)^i_M - \sum_{M=1}^{N} (M-1)T^{M-2}(p/f)^i_M}$$

$$= \frac{V_N[1 - S(p/f)^i_N]N}{(N+1)(p/a)^i_N - \sum_{M=1}^{N} M(p/f)^i_M} \qquad (30)$$

The summation $\sum_{M=1}^{N} M(p/f)^i_M$ can be written as:

$$\sum_{M=1}^{N} M(p/f)^i_M = \sum_{M=1}^{N} Mq^{-M}$$

$$= q^{-1} + 2q^{-2} + 3q^{-3} + \ldots + Nq^{-N}$$

$$= (p/a)^i_N + (p/g)^i_N \qquad (31)$$

Substituting the value of the sum from Equation 31 into Equation 30, R_1 becomes:

$$R_1 = \frac{V_N[1 - S(p/f)^i_N]}{(p/a)^i_N - N^{-1}(p/g)^i_N} \qquad (32)$$

Now, recognizing that when T = 1, the operation returns will be declining each period by a constant amount equal to (R_1/N), R_X can be expressed as:

$$R_X = R_1 - (X-1)\left(\frac{R_1}{N}\right)$$

$$= R_1\left[\frac{N - X + 1}{N}\right] \tag{33}$$

Substituting the value of R_1 from Equation 32 into Equation 33, the operation return ratio for the case when T = 1 will be:

$$\frac{R_X}{V_N} = \left[\frac{1 - S(p/f)_N^i}{(p/a)_N^i - N^{-1}(p/g)_N^i}\right]\left[\frac{N - X + 1}{N}\right] \tag{34}$$

It should be noted that the value of the property will be computed by still using Equation 11 or 12 provided that C_X' is calculated by using Equation 29.

<u>The case when T = ∞</u>

Recalling Equation 1, R_X can be expressed as follows when $T = \infty$:

$$R_X = \lim_{T\to\infty} R_1\left[\frac{NT^{N-1} - (X-1)T^{X-2}}{NT^{N-1}}\right]$$

$$= \lim_{T\to\infty} R_1\left[1 - \left(\frac{X-1}{N}\right)\left(\frac{1}{T^{N-X+1}}\right)\right]$$

$$= R_1 \tag{35}$$

and the operation return will be constant irrespective of age. To evaluate this constant quantity, R_1, Equation 5 will be used:

$$R_1 = \lim_{T\to\infty} \frac{V_N[1 - S(p/f)_N^i]\ NT^{N-1}}{\sum_{M=1}^{N} [NT^{N-1} - (M-1)T^{M-2}](p/f)_M^i}$$

$$= \lim_{T\to\infty} \frac{V_N[1 - S(p/f)_N^i]}{\sum_{M=1}^{N} \left[1 - \left(\frac{M-1}{N}\right) T^{M-N-1}\right](p/f)_M^i}$$

In the last equation, (M-N-1) will always be negative and R_1 can then be written as:

$$R_1 = V_N[1 - S(p/f)_N^i](a/p)_N^i \tag{36}$$

The operation return ratio can then be expressed as:

$$\frac{R_X}{V_N} = \frac{R_1}{V_N} = [1 - S(p/f)_N^i](a/p)_N^i$$

$$= (1-S)(a/p)_N^i + S[1-(p/f)_N^i](a/p)_N^i$$

$$= (1-S)(a/p)_N^i + iS \tag{37}$$

To derive an expression for C_X' when $T = \infty$, the present worth principle will be applied at any age X:

$$V_X = R_X(p/a)^i_{N-X} + V_S(p/f)^i_{N-X}$$

$$= V_N[(1-S)(a/p)^i_N + iS](p/a)^i_{N-X} + V_S(p/f)^i_{N-X}$$

$$= [V_{ND}(a/p)^i_N + iV_S](p/a)^i_{N-X} + V_S(p/f)^i_{N-X}$$

$$= V_{ND}(a/p)^i_N(p/a)^i_{N-X} + V_S[i(p/a)^i_{N-X} + (p/f)^i_{N-X}]$$

$$= V_{ND}\ C'_X + V_S \tag{38}$$

where C'_X is the condition percent factor when $T = \infty$ and can be expressed as:

$$C'_X = (a/p)^i_N(p/a)^i_{N-X}$$

$$= \frac{(1+i)^N - (1+i)^X}{(1+i)^N - 1} \tag{39}$$

Equations 38 and 39 for the value and the condition percent factor for the special case of $T = \infty$ are the same equations derived by Winfrey for the uniform operation return case (5, p. 25). In other words, the model presented here reduces to Winfrey's model when the progression rate tends to infinity.

When the modified condition percent factor is multiplied by 100, it is called the "modified condition percent." Tables giving Winfrey's condition percent for different lives and discount rates are already published (13).

The case when i = 0

When the discount rate is zero, $(p/f)_M^i$ will equal unity and Equation 13 reduces to:

$$C_X' = \frac{(N-X)\ T^{N+1} - T^X \sum_{M=1}^{N-X} T^M}{N\ T^{N+1} - \sum_{M=1}^{N} T^M} \qquad (40)$$

Applying Equation 20 to the two sums in Equation 40, C_X' can be expressed as:

$$C_X' = \frac{(N-X)\ T^{N+1} - T^{X+1}(T^{N-X} - 1)/(T-1)}{NT^{N+1} - T(T^N - 1)/(T-1)}$$

$$= \frac{T^{N+1}[(T-1)(N-X)-1] + T^{X+1}}{T^{N+1}[(T-1)N - 1] + T} \qquad (41)$$

The operation return ratio can be evaluated when i = 0 by first recalling Equation 5 and substituting $(p/f)_M^i$ by 1. R_1 can then be written as:

$$R_1 = \frac{V_N(1-S)(T^N-1)}{\sum_{M=1}^{N} T^N - T^{-1} \sum_{M=1}^{N} T^M}$$

$$= \frac{V_N(1-S)(T^N-1)}{NT^N - (T^N - 1)/(T - 1)}$$

$$= \frac{V_N(1-S)(T^N-1)(T-1)}{T^N(NT - N - 1) + 1} \qquad (42)$$

and the operation return ratio will be given by:

$$\frac{R_X}{V_N} = \frac{R_1}{V_N}\left[\frac{T^N - T^{X-1}}{T^N - 1}\right]$$

$$= \frac{(1-S)(T-1)(T^N - T^{X-1})}{T^N(NT - N - 1) + 1} \qquad (43)$$

The value of the property at age X for the case when $i = 0$ will still be given by Equation 11 or 12 provided C_X' is evaluated by using Equation 41.

The case when $T = 1$ and $i = 0$

The modified condition percent factor can be derived in this case by evaluating Equation 40 when $T = 1$ as follows:

$$C_X' = \frac{(N-X)\ T^{N+1} - T^X \sum_{M=1}^{N-X} T^M}{NT^{N+1} - \sum_{M=1}^{N} T^M} \tag{40}$$

$$= \lim_{T \to 1} \frac{(N-X)(N+1)T^N - \sum_{M=1}^{N-X} T^{M+X-1}\ (M+X)}{N(N+1)\ T^N - \sum_{M=1}^{N} MT^{M-1}}$$

$$= \frac{(N-X)(N+1) - \sum_{M=1}^{N-X} M - \sum_{M=1}^{N-X} X}{N(N+1) - \sum_{M=1}^{N} M}$$

$$= \frac{(N-X)(N+1) - 0.5(N-X)(N-X+1) - X(N-X)}{N(N+1) - 0.5\ N(N+1)}$$

$$= \frac{(N-X)(N-X+1)}{N(N+1)} \tag{44}$$

As for deriving an expression for the operation return ratio, $(p/f)_N^i$ will be substituted by 1 in Equation 5 and then the limit will be taken as T tends toward 1.

$$R_1 = \frac{V_N[1-S(p/f)_N^i][T^N - 1]}{\sum_{M=1}^{N} [T^N - T^{M-1}](p/f)_M^i} \tag{5}$$

$$= \lim_{T \to 1} \frac{V_N [1-S] \; N \; T^{N-1}}{N \sum_{M=1}^{N} T^{N-1} - \sum_{M=1}^{N} (M-1) \; T^{M-2}}$$

$$= \frac{V_N \; N \; (1-S)}{N^2 - \sum_{M=1}^{N} M + \sum_{M=1}^{N} 1} = \frac{V_N \; N(1-S)}{N^2 - 0.5N(N+1)+N}$$

$$= \frac{2 \; V_N \; (1-S)}{N + 1} \qquad (45)$$

In this case, R_X can be written as:

$$R_X = R_1 \left[\frac{N - X + 1}{N}\right] \qquad (33)$$

$$= \left[\frac{2 \; V_N \; (1-S)}{N + 1}\right] \left[\frac{N - X + 1}{N}\right]$$

and the operation return ratio when T = 1 and i = 0 will be:

$$\frac{R_X}{V_N} = \frac{2(1-S)(N-X+1)}{N(N+1)} \qquad (46)$$

The value of the property at age X in this case can be expressed by substituting $(p/f)_N^i$ and $(p/f)_{N-X}^i$ by 1 in Equations 11 and 12. The resulting equations will be:

$$V_X = V_N[C_X' \ (1-S) + S] \tag{47}$$

$$= V_{ND} \ C_X' + V_S \tag{48}$$

and, of course, C_X' is given by Equation 44 in this case.

The case when $T = \infty$ and $i = 0$

In this case, C_X' will be derived from Equation 39 as follows:

$$C_X' = \lim_{i \to 0} \frac{N(1+i)^{N-1} - X(1+i)^{X-1}}{N(1+i)^{N-1}}$$

$$= \frac{N - X}{N} \tag{49}$$

and the operation return ratio can be derived from Equation 37 by substituting i by zero and $(a/p)_N^i$ by $(1/N)$. It will be:

$$\frac{R_X}{V_N} = \frac{1-S}{N} \tag{50}$$

Again, Equations 11 and 12 are applicable for computing V_X and in this case will reduce to:

$$V_X = V_N[C_X' \ (1-S) + S] \tag{51}$$

$$= V_{ND} \ C_X' + V_S \tag{52}$$

where C_X' is computed from Equation 49.

To summarize all of the above results, the different expressions for the modified condition percent factor, the operation return ratio and the value will be rewritten below. This will be for the general case as well as the special cases.

<u>The general case</u>

$$C_X' = \frac{T^{N+1} \sum_{M=1}^{N-X} (p/f)_M^i - T^X \sum_{M=1}^{N-X} T^M (p/f)_M^i}{T^{N+1} \sum_{M=1}^{N} (p/f)_M^i - \sum_{M=1}^{N} T^M (p/f)_M^i} \tag{13}$$

$$= \frac{q^{N-X-1} (T + iT^{X-N}) - q^{-1}(T + i) - q^{N-X} + 1}{q^{N-X-1} (T + iT^{-N}) - q^{-X-1}(T+i) - q^{N-X} + q^{-X}} \tag{26}$$

$$\frac{R_X}{V_N} = \frac{[1-S(p/f)_N^i][T^N - T^{X-1}]}{\sum_{M=1}^{N} T^N (p/f)_M^i - T^{-1} \sum_{M=1}^{N} T^M (p/f)_M^i} \tag{6}$$

$$V_X = V_N \left[C_X'(1-S) + S\left[\{1 - (p/f)_N^i\}C_X' + (p/f)_{N-X}^i\right]\right] \tag{11}$$

$$= V_{ND}\, C_X' + V_S\left[\{1 - (p/f)_N^i\}C_X' + (p/f)_{N-X}^i\right] \tag{12}$$

The case when T = 1

$$C_X' = \frac{q^{N-X}(1 + iX - iN) - 1}{q^{N-X}(1 - iN) - q^{-X}} \tag{29}$$

$$\frac{R_X}{V_N} = \left[\frac{1 - S(p/f)\frac{i}{N}}{(p/a)\frac{i}{N} - (1/N)(p/g)\frac{i}{N}}\right]\left[\frac{N - X + 1}{N}\right] \tag{34}$$

$$V_X = V_N\left[C_X'(1-S) + S\left[\{1-(p/f)\frac{i}{N}\} C_X' + (p/f)\frac{i}{N-X}\right]\right] \tag{11}$$

$$= V_{ND}\, C_X' + V_S\left[\{1 - (p/f)\frac{i}{N}\} C_X' + (p/f)\frac{i}{N-X}\right] \tag{12}$$

The case when T = ∞

$$C_X' = \frac{(1+i)^N - (1+i)^X}{(1+i)^N - 1} \tag{39}$$

$$\frac{R_X}{V_N} = (1-S)(q/p)\frac{i}{N} + iS \tag{37}$$

$$V_X = V_{ND}\, C_X' + V_S \tag{38}$$

$$= V_N[(1-S)\, C_X' + S]$$

The case when $i = 0$

$$C_X' = \frac{T^{N+1}[(T-1)(N-X)-1]+T^{X+1}}{T^{N+1}[(T-1)N-1] + T} \tag{41}$$

$$\frac{R_X}{V_N} = \frac{(1-S)(T-1)(T^N - T^{X-1})}{T^N(NT-N-1) + 1} \tag{43}$$

$$V_X = V_{ND}\, C_X' + V_S \tag{38}$$

$$= V_N[(1-S)C_X' + S]$$

The case when $T = 1$ and $i = 0$

$$C_X' = \frac{(N-X)(N-X+1)}{N(N+1)} \tag{44}$$

$$\frac{R_X}{V_N} = \frac{2(1-S)(N-X+1)}{N(N+1)} \tag{46}$$

$$V_X = V_{ND}\, C_X' + V_S \tag{48}$$

$$= V_N[(1-S)\, C_X' + S]$$

The case when $T = \infty$ and $i = 0$

$$C_X' = \frac{N-X}{N} \tag{49}$$

$$\frac{R_X}{V_N} = \frac{1-S}{N} \tag{50}$$

$$V_X = V_{ND} C_X' + V_S \tag{52}$$

$$= V_N[(1-S) C_X' + S]$$

From the foregoing developments, it is seen that the proposed model represents a multitude of situations. Progression rates ranging from just above zero to infinity can be represented. At the same time discount rates of zero or larger can be applied. Also, salvage ratios can be positive, negative or zero.

MODEL CHARACTERISTICS

It is of interest, after the model has been developed, to investigate its characteristics. The effects of change of parameters such as the progression rate, discount rate, probable life, average service life or salvage on the results will be shown and discussed.

It should be pointed out that the model can be applied to units of property as well as to groups of property units. The case of "mass accounts" held for several property groups can also be handled.

The following sections will illustrate the model characteristics for both cases of units and groups of property.

Property Units

The analyst would normally be interested in the operation return ratio (R_X/V_N), the condition percent factor C'_X and the value of property, V_X, through its life. After estimating the unit's probable life, he would estimate a discount rate or rate of return commensurate with the future operation of the property under study. The next task would be to apply judgment in choosing the progression rate so that the computed operation return ratios would match or closely approximate the most probable actual ones.

Operation return ratio characteristics

To show the effect of the progression rate on the operation return ratio, Figure 10 is presented. A unit with 10 years probable life was assumed to have zero salvage ratio. An annual discount rate of 6% or semi-annual discount rate of 2.956% was applied for purposes of illustration. In the figure, T ranges from 0.5 to infinity and it is seen that (R_X/V_N) is constant through the unit's life for $T = \infty$. For $T = 1$, (R_X/V_N) is a straight line. It is also shown that the operation return ratio is more sensitive to changes in the progression rate, T, for $T < 1$ than to changes in T when $T > 1$. For example, this can be shown clearly by observing the curves of (R_X/V_N) for $T = 0.7$, $T = 1.0$ and $T = 1.3$. It should be pointed out that the curves shown are the paths connecting points computed at the discrete ages of 1/2, 1, 1-1/2, 2, ... etc.

Figure 11 shows the effect of changing the discount rate on the operation return ratio. The effective annual discount rate was changed from zero to 14% for specific progression rate of 1.3 and salvage ratio of zero. It is seen that the operation return ratio increases with the increase of the discount rate, other parameters kept constant. This can be thought of that, the higher the discount rate, the higher will be the required periodic operation returns so that when discounted, they will equal the same value new, V_N.

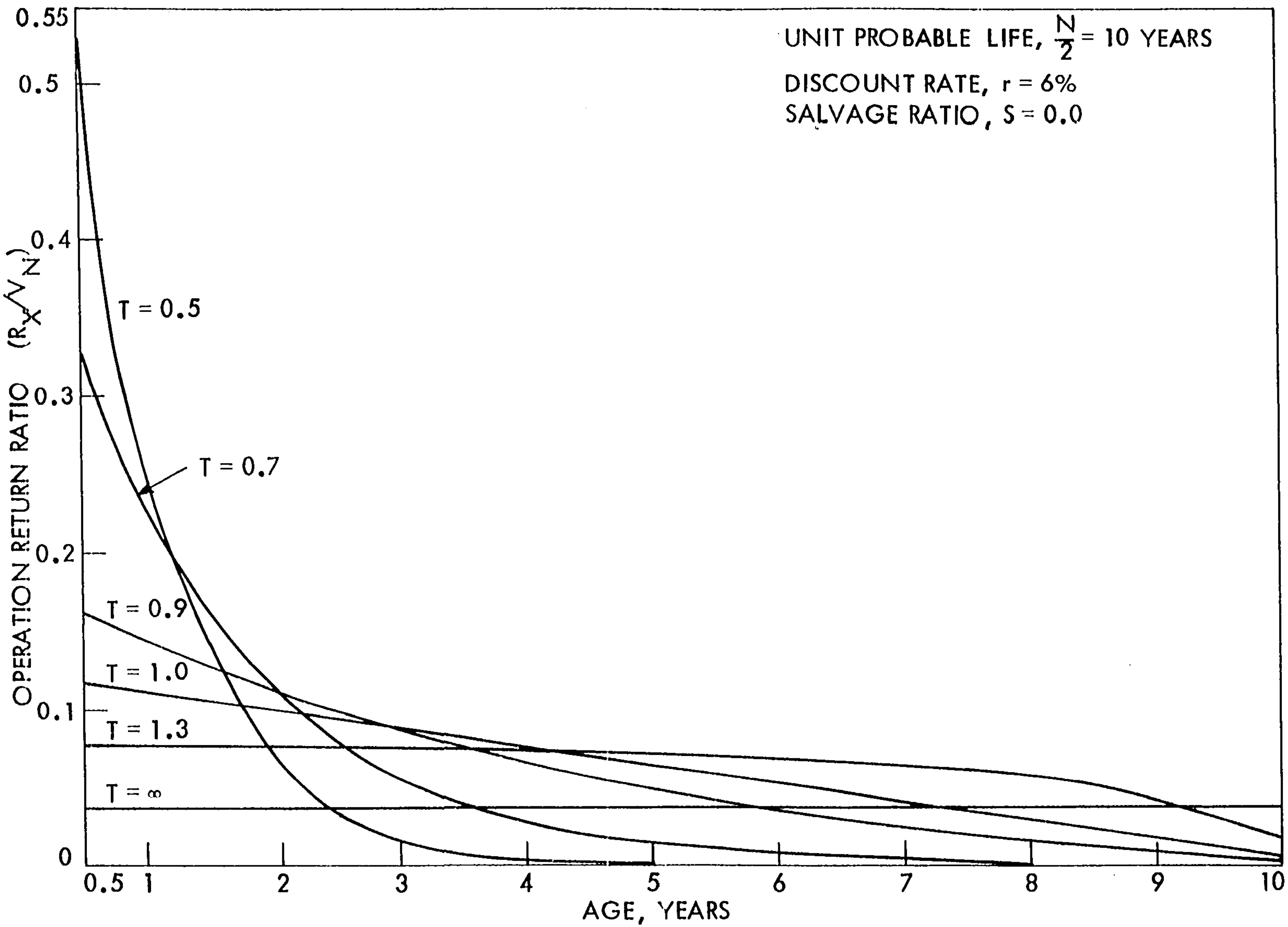

Fig. 10. Variation of unit operation return ratio with age for different progression rates

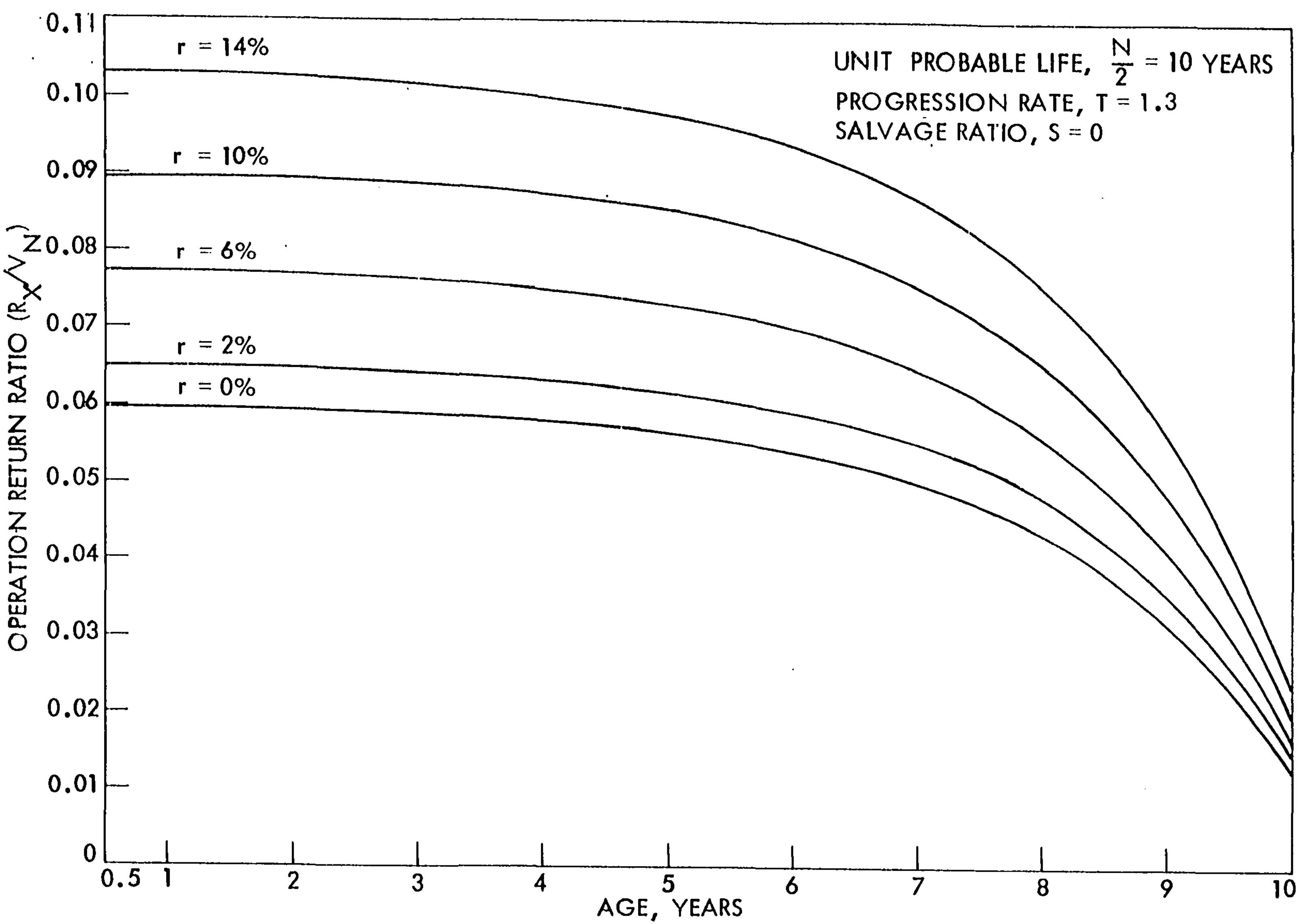

Fig. 11. Variation of unit operation return ratio with age for various discount rates

The effect of the salvage ratio on the operation return ratio is shown in Figure 12. It is shown that there is an inverse relationship between the salvage ratio and the operation return ratio, i.e. the higher the salvage ratio, the lower the required periodic operation return ratios will be. This is explained by restating that the operation return for a certain period is the return of investment plus the return on the unrecovered investment during that period. The result is that as S increases, the portion of the total investment to be recovered per period decreases and consequently the operation return ratio per period decreases.

Table A-1 in the Appendix shows the numerical results pertaining to some of the curves shown in Figures 10 and 11 while Table A-2 shows results for curves of Figure 12.

Modified condition percent characteristics

The variation of the modified condition percent with age for various progression rates for a sample unit of property is shown in Figure 13. It is seen that the condition percent factor varies widely with the progression rate and that, like the operation return ratio, it is more sensitive to changes of T for low T values than to similar changes for high T values. This can be clearly seen, e.g., by comparing the curves when T takes the values 0.3, 1.0, 1.3 and 2.0.

Figure 14 shows the effect of changing the discount rate r on the modified condition percent. The higher the discount

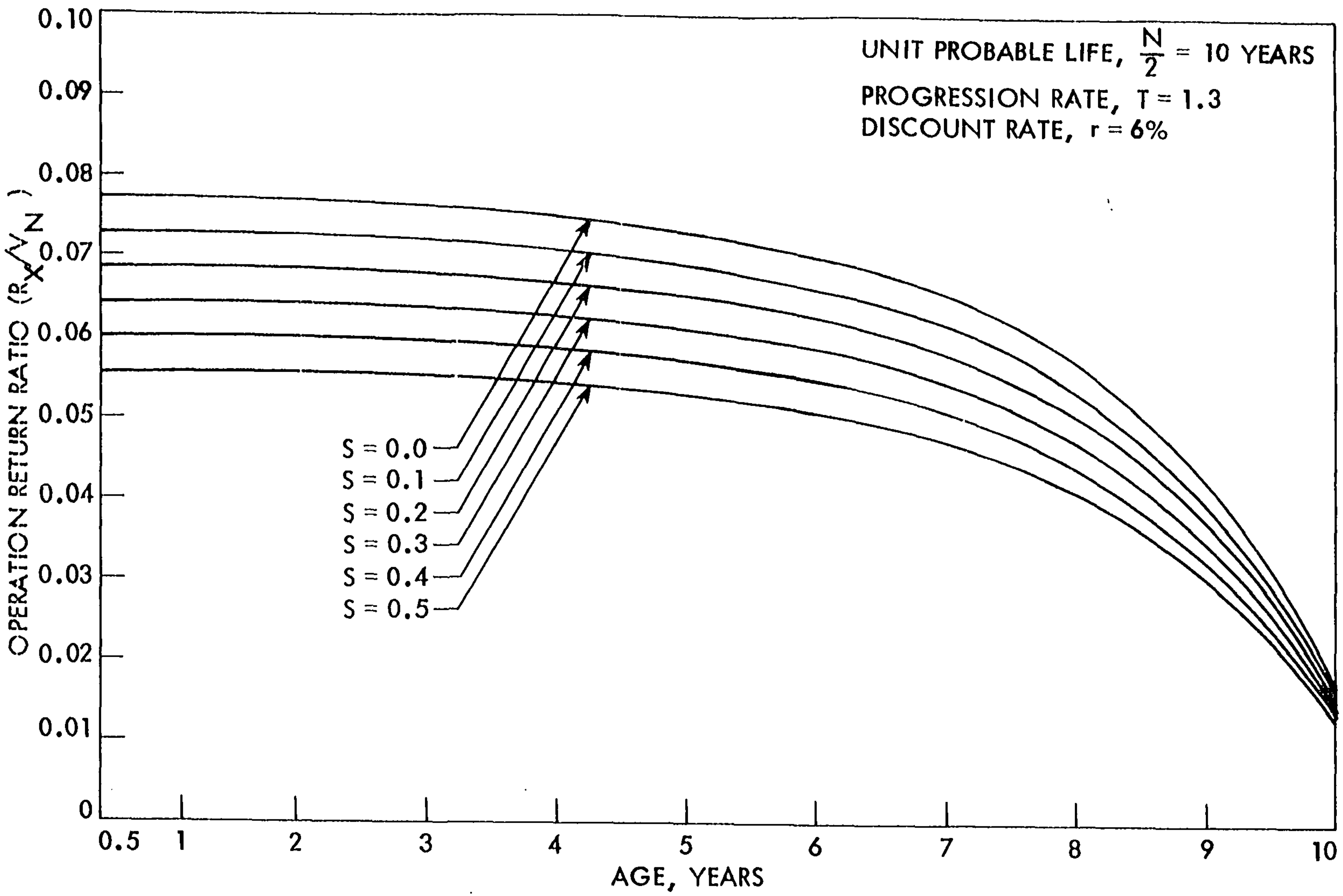

Fig.12. Variation of unit operation return ratio with age for various salvage ratios

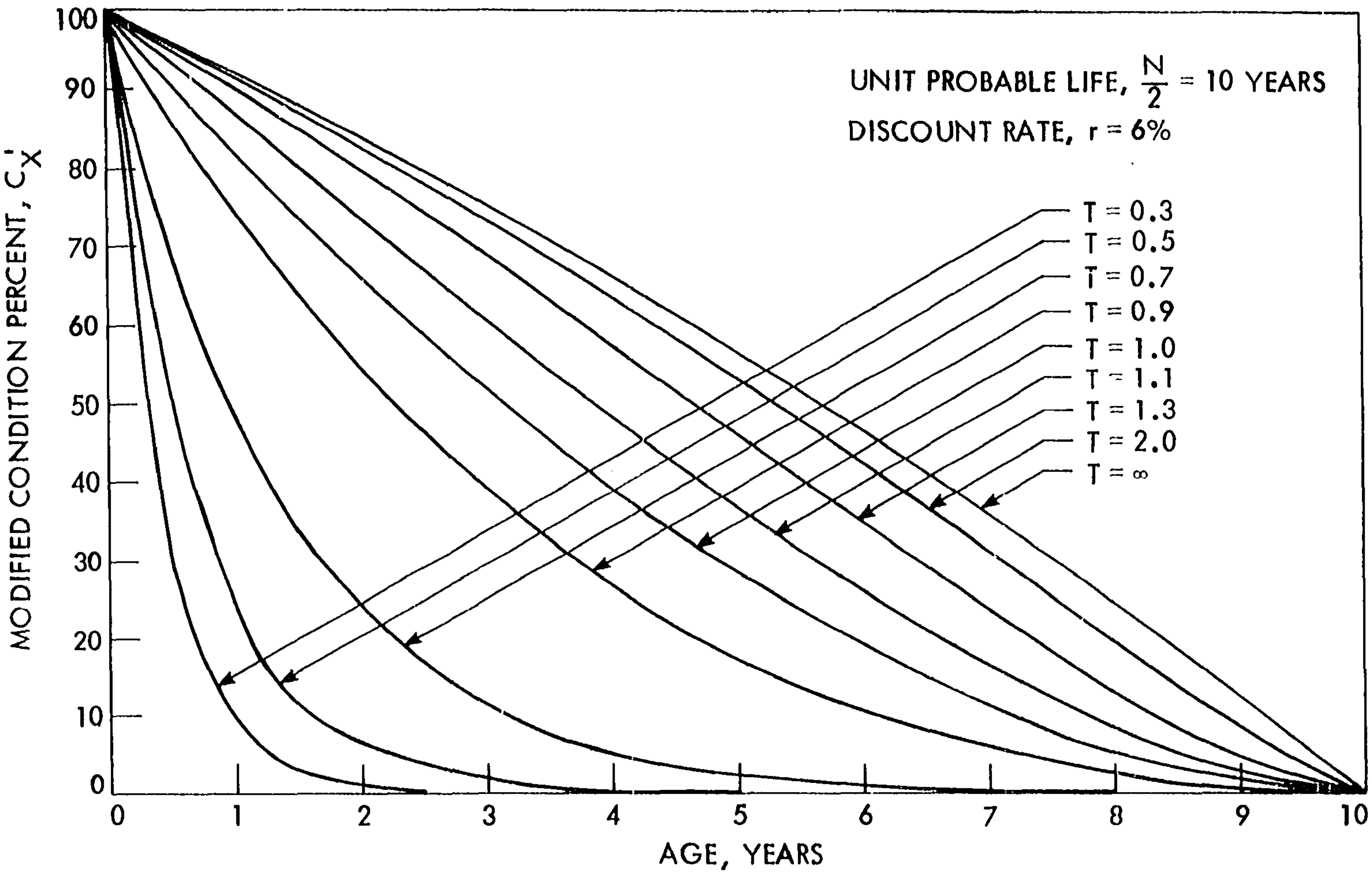

Fig. 13. Variation of unit modified condition percent with age for various progression rates

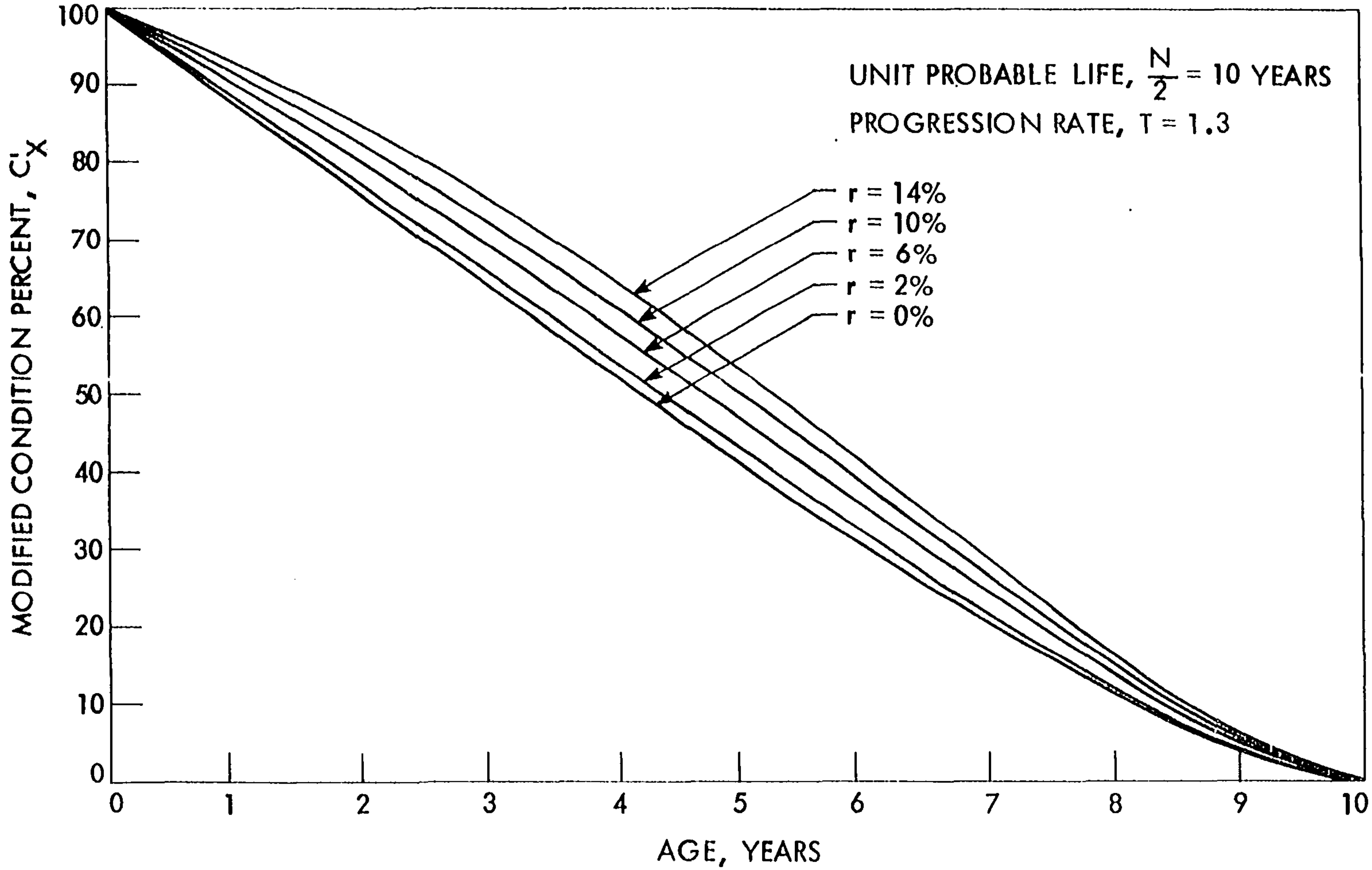

Fig. 14. Variation of unit modified condition percent with age for various discount rates

rate, the higher the modified condition percent.

Comparing the effects of the progression rate and the discount rate on the modified condition percent, it is clear from Figures 13 and 14 that, in general, the progression rate has more dramatic effect than do the discount rate.

The effect of the unit's probable life on the modified condition percent was investigated. For comparison purposes the age of the unit was generalized as a percentage of the probable life. Unit probable lives of 10, 20 and 30 years were used and T was assigned the values of 0.9, 1.0 and 2.0. Figure 15 shows three sets of curves. Each set corresponds to a certain value of T. It is observed from the figure that, in general, the probable life has an effect on the modified condition percent. It is also observed that a change in the probable life does not necessarily cause a change in the modified condition percent in the same direction every time. For example, it is seen from Figure 15 that for T = 0.9, C_X' is larger when the probable is 10 years than when it is 20 or 30 years for corresponding ages as percents of probable lives. On the other hand, for T = 1.0 and T = 2.0 the situation is the reverse.

Table A-3 in the Appendix gives sample numerical results for some of the curves shown in Figures 13 and 14. Table A-4 gives sample results for curves shown in Figure 15.

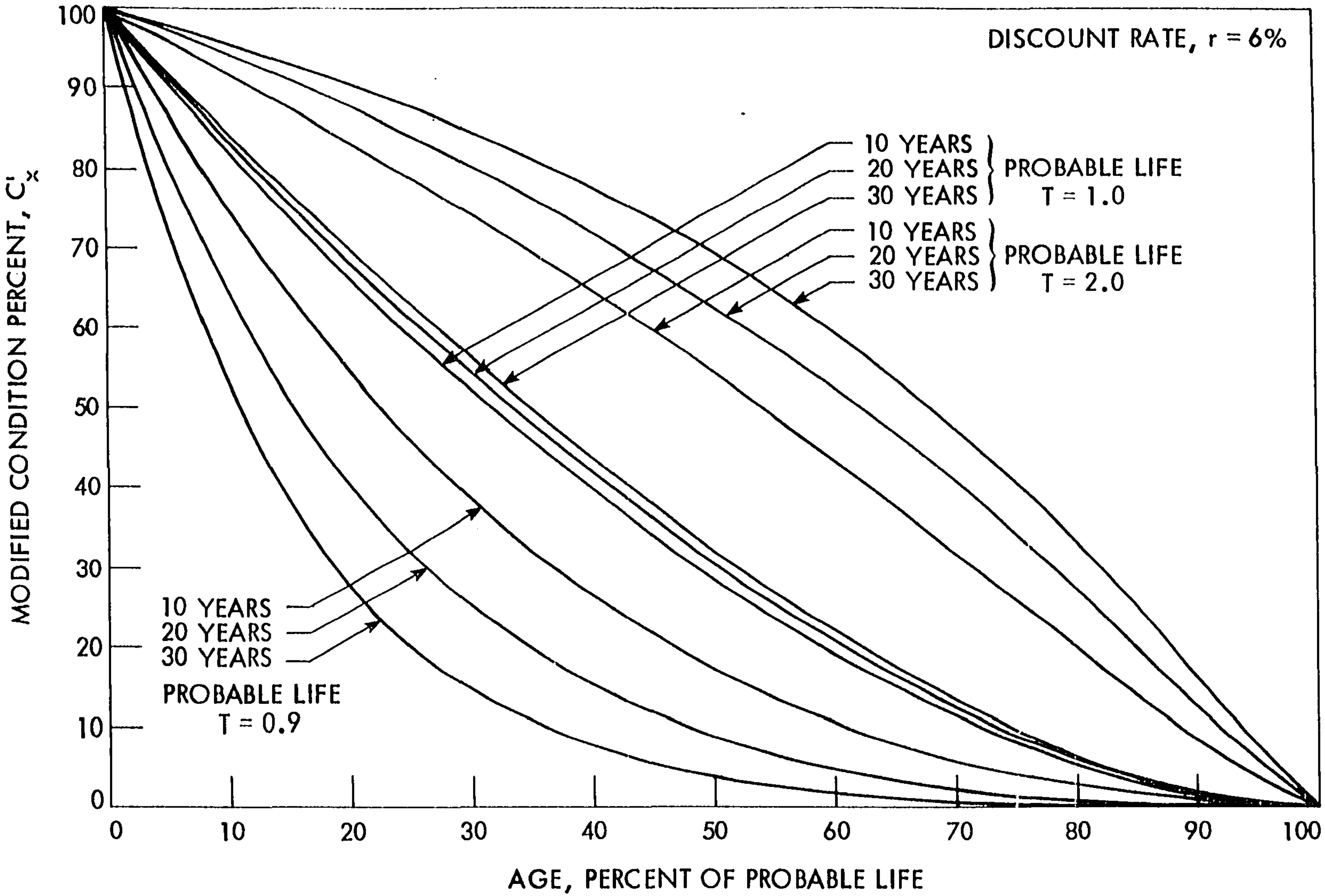

Fig. 15. Variation of modified condition percent with probable life of property units

Groups of Property Units

The model is applied to property groups by the use of the "Unit-summation procedure" advanced by Winfrey (5). It is called "Equal life group method" alternately in the literature (32, 33).

The basic concept of this procedure is to separate the surviving units comprising the group into frequency groups of units of like probable life as predicted from a forecasted retirement dispersion pattern. Since the units of a frequency group all have the same expected life, they can be treated in total as a single item having that life.

The modified condition percent factor for all the survivors of the group at the given age is then obtained by weighting each modified condition percent factor, calculated on an item basis for each frequency group surviving, by the number of units in each frequency group. The final result is the weighted average modified condition percent factor of the survivors of the group. As an illustration, Figure 16 shows, for an S_0 Iowa type curve and 5 years average service life, how the survivors are segregated into frequency groups with assumed common probable lives. In this example, the progression rate was assumed to equal unity and it is applied for every frequency group separately as shown.

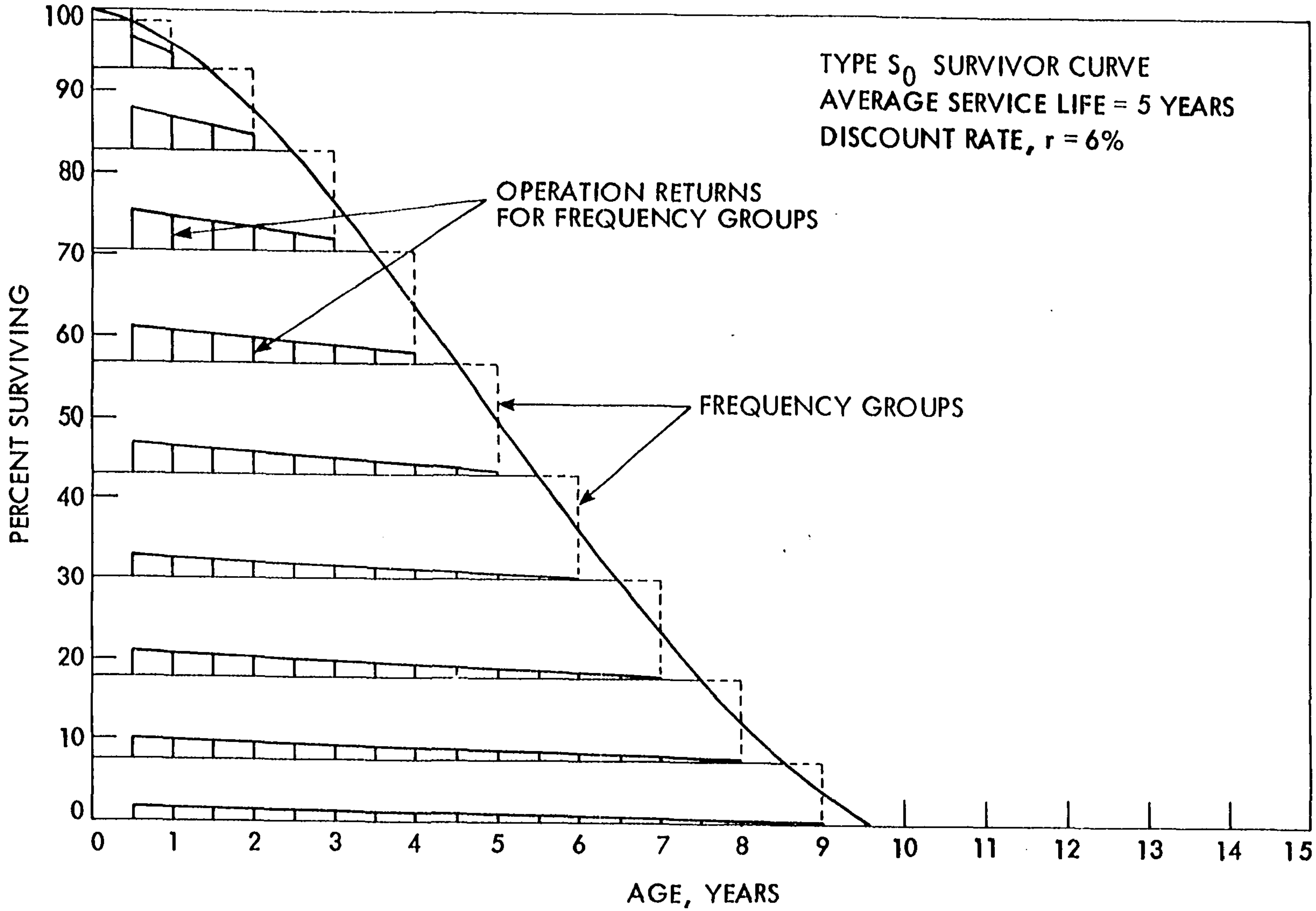

Fig. 16. Application of unit summation procedure to declining operation returns case

Modified condition percent characteristics

The effect of the progression rate, T, on the modified condition percent, C_X', for a group of property units is illustrated by Figure 17. Similar to the property unit situation, C_X' increases with the increase of T for the same age. Figure 18 shows the effect of the discount rate, r, and consequently i, on C_X'. Here again, C_X' increases with the increase of r. However, it is seen that, in general, C_X' is more sensitive to the change of T than to the change of r.

Figure 19 illustrates the effect of the retirement dispersion of the group on the modified condition percent. The more dispersed the retirements of a group, like that following an S_0 type curve, the higher will be its C_X' at later ages of the group than for a group with less retirement dispersion, e.g. an S_2 type curve. This is due to the fact that the group following the S_2 type dispersion has more surviving units before average service life and less surviving units after average service life than the group following the S_0 type dispersion.

Figure 20 shows a sample application to three selected type curves. In this case, the curves for the S_2 and R_2 dispersions were similar. The curve for the L_2 dispersion showed higher C_X' for later ages of the group due to the much longer maximum life.

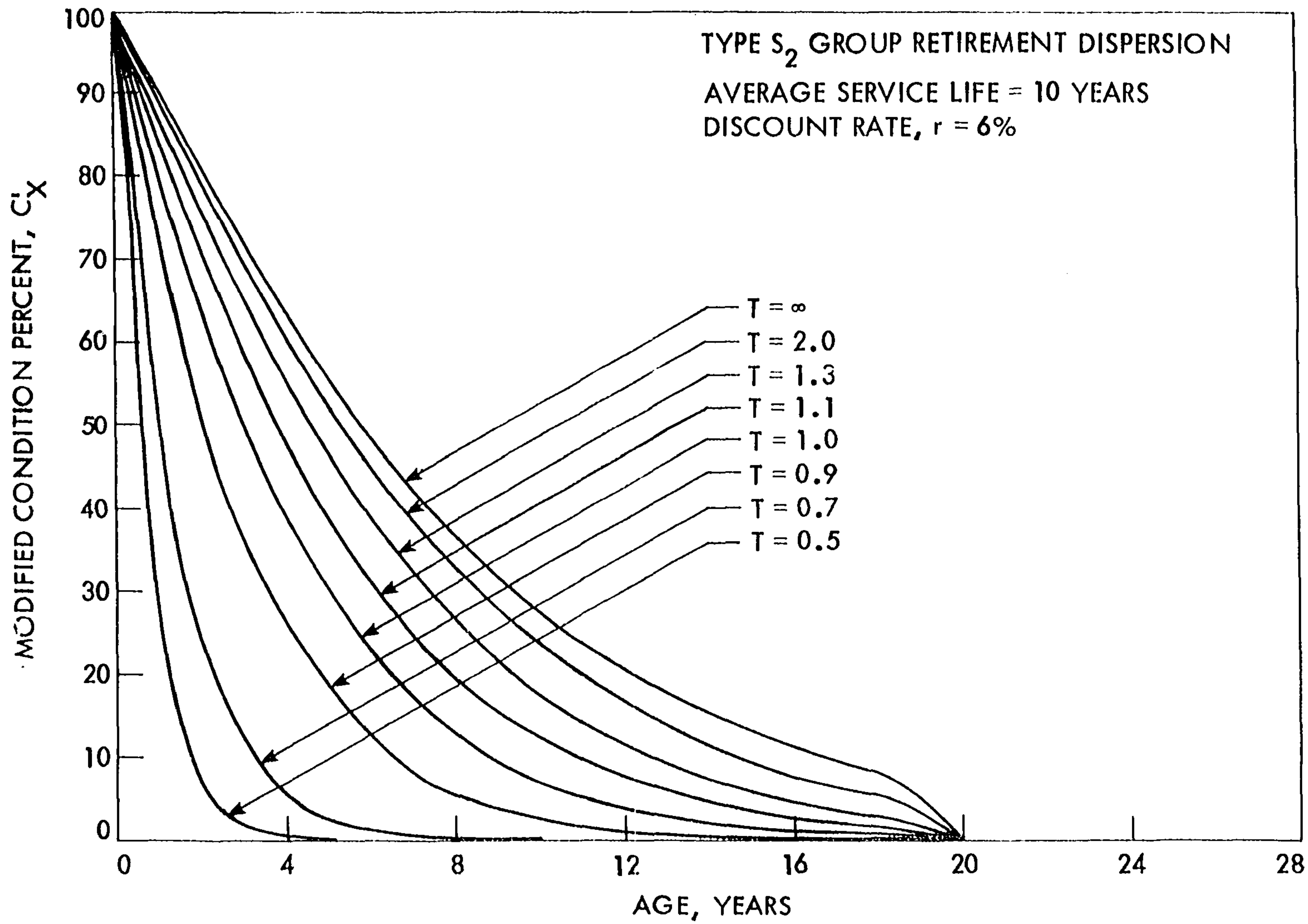

Fig. 17. Effect of progression rate on modified condition percent for a group

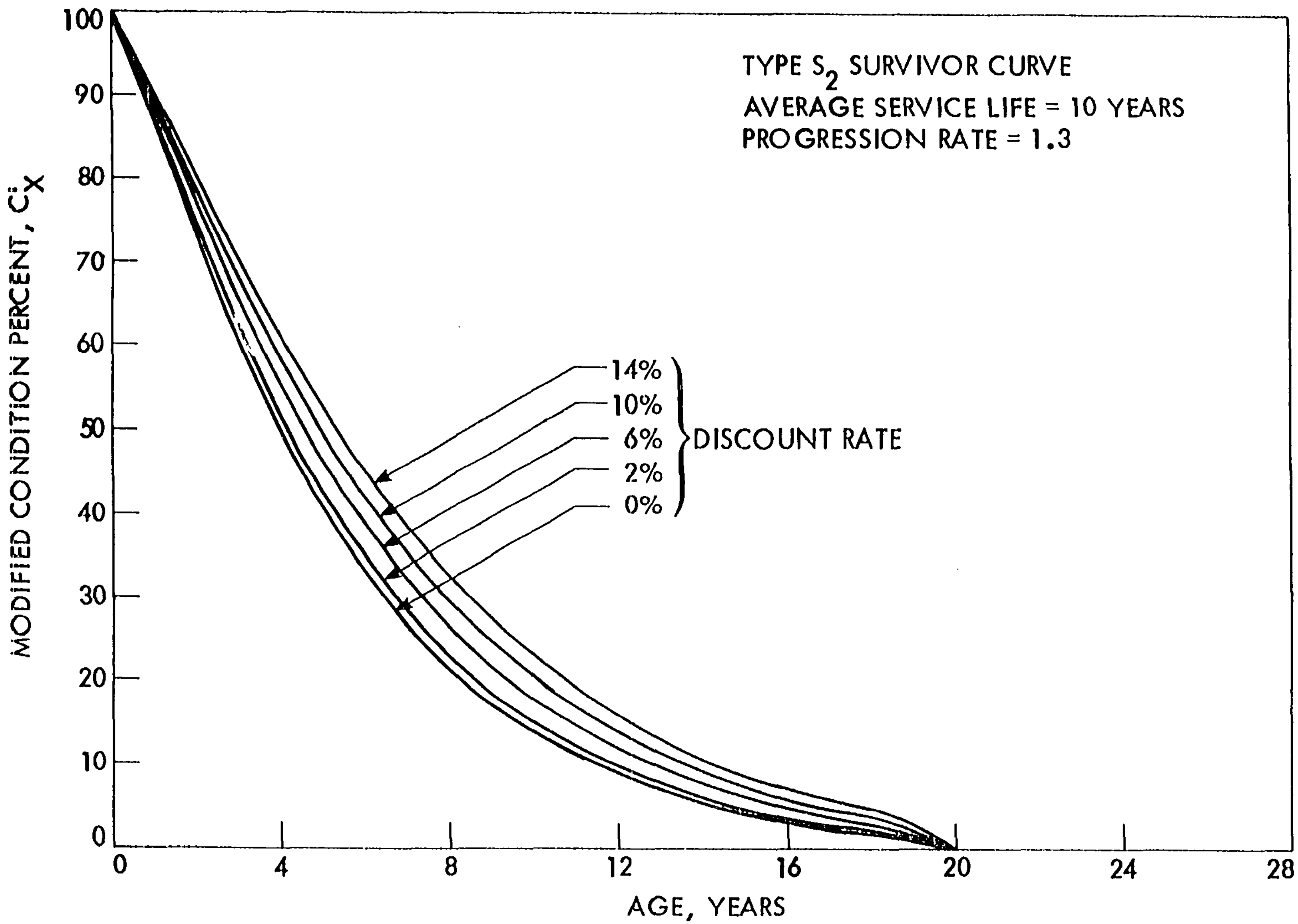

Fig. 18. Effect of discount rate on modified condition percent for a group

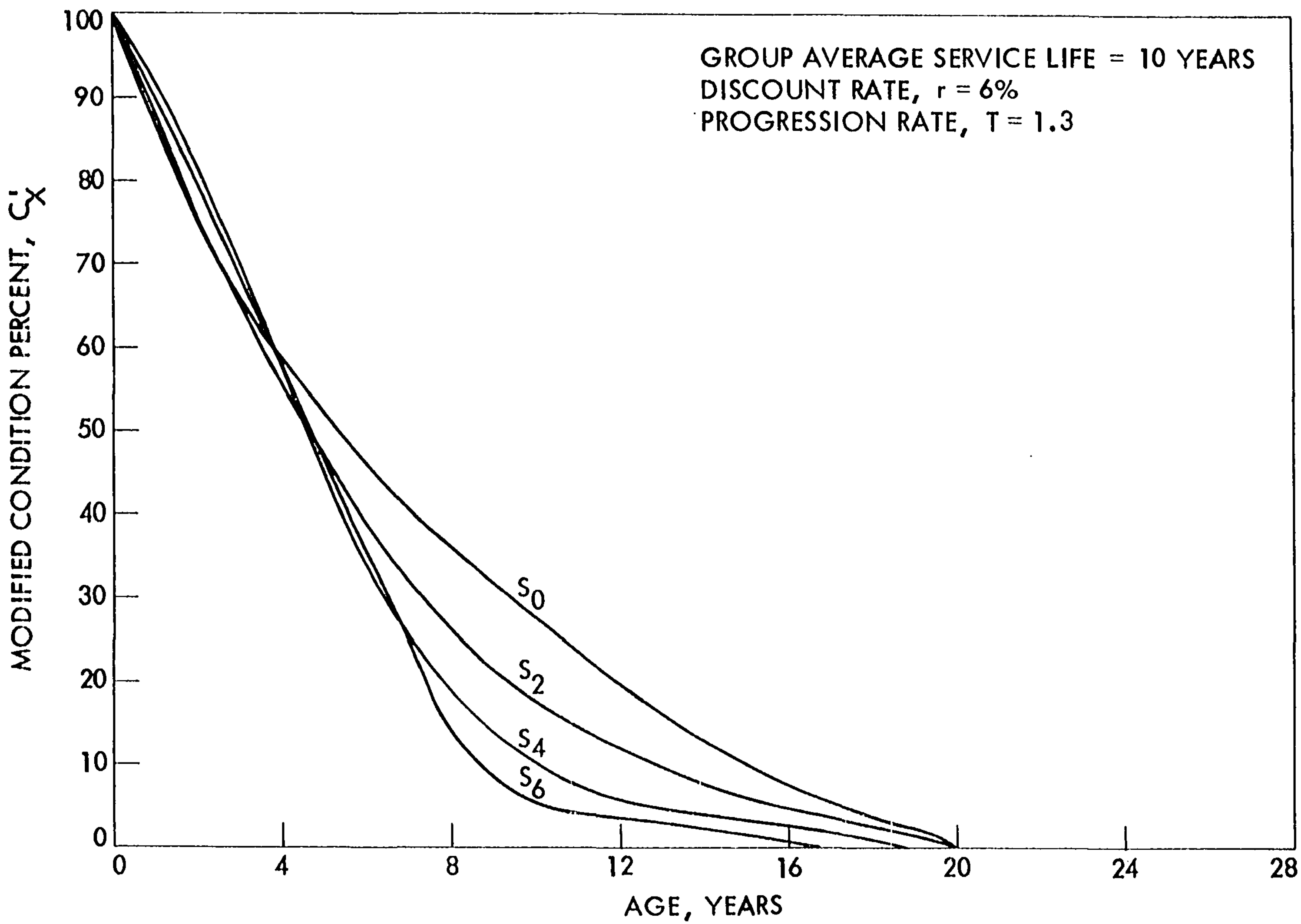

Fig. 19. Variation of modified condition percent with age for four symmetrical type curves

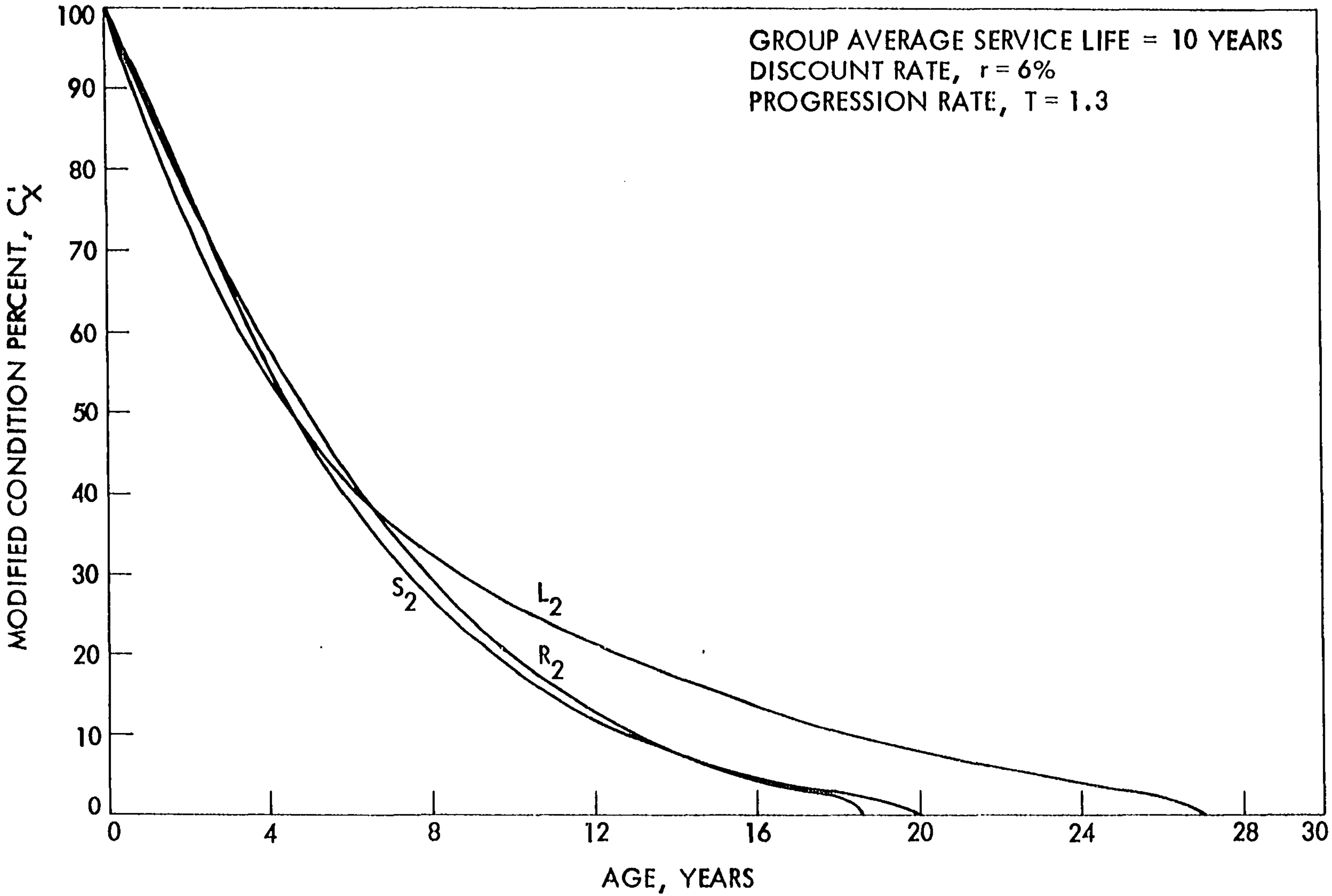

Fig. 20. Variation of modified condition percent with age for selected type curves

The effect of the average service life on the modified condition percent for a property group is illustrated in Figures 21 and 22. All other parameters were kept the same for both figures except the progression rate and C_X' was computed for average service lives of 5, 10 and 15 years. Figure 21 shows that, for comparable lives and for T = 1.3, C_X' increases as the average service life increases. On the other hand, for T = 0.9, Figure 22 shows that, for comparable lives, C_X' decreases as the average service increases. These observations are in line with what was observed for units of property and shown in Figure 15. It is noted that Figures 21 and 22 could have been drawn for generalized ages as percent of average service life as was done for property units in Figure 15.

Numerical results pertaining to Figure 17 are given in Table A-5 in the Appendix. Differences will be found between modified condition percents for property groups when $T = \infty$, e.g. like those given in Table A-5, and condition percents given by Winfrey for the same parameters (13). Although the same equation is used, Winfrey calculates the probable life of each frequency group by dividing each frequency group into 100 subgroups and then calculate their weighted probable life which is used as the probable life of the frequency group. In the calculations presented here, the probable life is assumed to be at the midyear when the frequency group will probably be

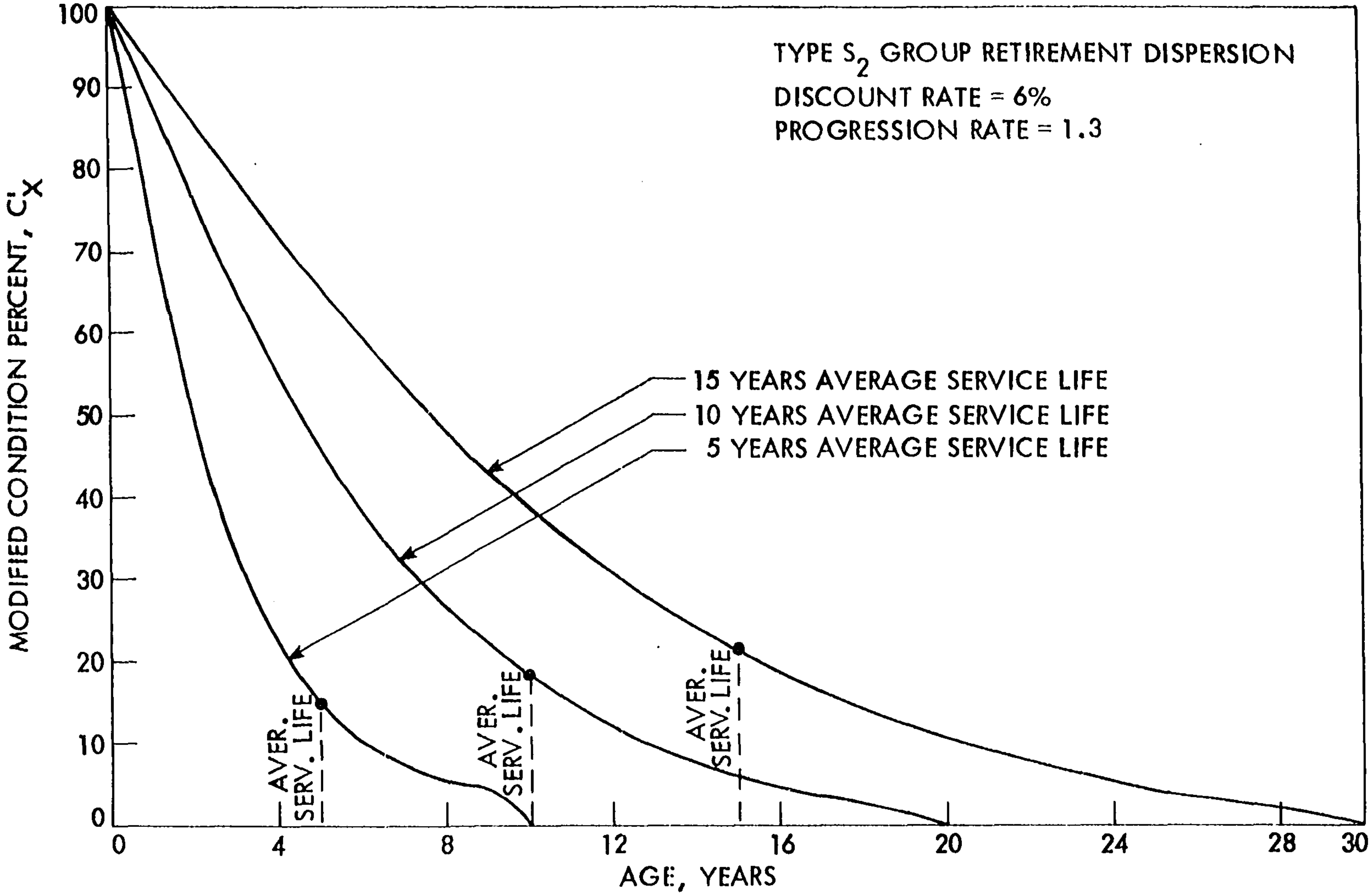

Fig. 21. Effect of average service life on modified condition percent for a group

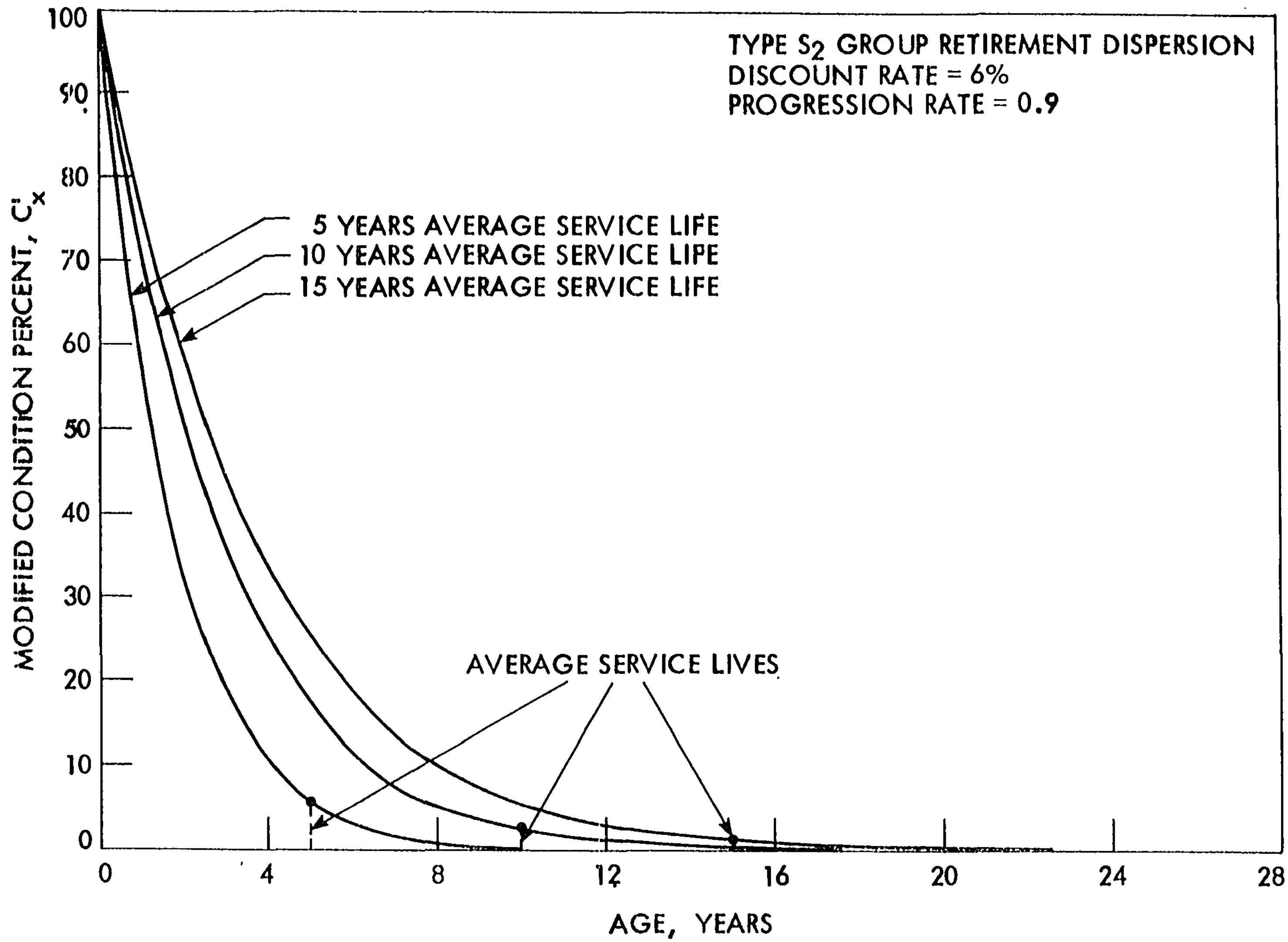

Fig. 22. Effect of average service life on modified condition percent for a group

retired. For example, if the frequency group will probably retire between ages 2-1/2 and 3-1/2 years, the probable life of this frequency group is assumed to be 3 years.

Tables A-6, A-7 and A-8 in the same appendix give results pertaining to Figures 18, 19 and 20 consecutively while Table A-9 refers to Figures 21 and 22.

Progression Rate

The progression rate, T, of the services or operation returns has a significant effect on the modified condition percent factors and property values estimated. It is felt that a range of 5-∞ for T may be suitable for properties with uniform or nearly uniform service through their lives, e.g., telephone poles in utilities. For properties which experience moderate obsolescence, deterioration and/or rising operating costs, a range of 1-5 for T may be appropriate. On the other hand, for properties subjected to the more severe forces of obsolescence, deterioration and service interruption, a range of 0.7-1.0 for T may be assigned. Examples of this last case would include electric generating units put on stand-by operation due to system growth, defense machinery and equipment, and properties affected severely by model changes and changes of the art.

EXPERIMENTAL PROCEDURES

Two sorts of data were sought for this study. The first is life analysis data from Iowa industries to test the reasonableness of utilizing the well-known Iowa type survivor curves in estimating values of machinery and equipment at different ages. The second is data relevant to the pattern of services derivable from the machinery and equipment through their lives.

Life Analysis Procedures

In general, data fit for life analysis can be actuarial or semi-actuarial. The actuarial data provide the installation date for each particular retirement and for each particular survivor (34, p. 11). The semi-actuarial data do not provide the age of plant retirements at the time of their retirements.

To show the difference between actuarial and semi-actuarial data, Table 1 will be used which shows hypothetical data for seven vintage groups. If aged data, or all the information in this table, is known, the data is called actuarial. On the other hand, if only the annual gross additions and the yearly plant balances or retirements are known, the data is called semi-actuarial. In this case, only the first four columns and the bottom row of Table 1 would be available. For example the different ages at retirement of

Table 1. Actuarial data for seven vintage groups

Year of placing	Total amount placed	Total amount of plant retired	Total amount of plant in service 1-1-68	Upper figure: Plant remaining in service at beginning of indicated calendar year Lower figure: Plant retired during indicated calendar year							
	Year of inventory:			61	62	63	64	65	66	67	68
61	10	10	0	$_{1}$	9_{2}	7_{3}	4_{2}	2_{1}	1_{1}		
62	15	13	2		$_{1}$	14_{2}	12_{3}	9_{3}	6_{1}	5_{3}	2
63	6	5	1				6	6_{1}	5_{2}	3_{2}	1
64	20	10	10					20_{2}	18_{4}	14_{4}	10
65	10	4	6						10_{1}	9_{3}	6
66	12	3	9						$_{1}$	11_{2}	9
67	12	0	12								12
Totals	85	45	40	$_{1}$	9_{3}	21_{5}	22_{5}	37_{7}	40_{10}	42_{14}	40

the 7 units retired during 1965 would not be known.

Companies contacted to provide data for life analysis were asked to provide actuarial data as a first preference. If the companies' records were not kept to provide actuarial data, semi-actuarial data were used.

Several recognized methods are available for analyzing actuarial and semi-actuarial data (23, 34). Of the actuarial methods, the retirement rate method is thought to be the best because it is based on the collection and compilation of the data of all property in service during a period of recent years, both property retired and that still in service (1, p. 154).

If only semi-actuarial data are available, the simulated plant-record method is thought to be the best for several reasons (34). It is suspected that the most important reason is that this method provides, for semi-actuarial data, not only the probable average service life but also the element of mortality dispersion for groups of property.

Accordingly, it was decided to use the retirement rate method for analyzing actuarial data and to use the simulated plant-record for the analyses of semi-actuarial data.

Retirement rate method

This method allows the use of an experience band which shows the experience or retirements that have occurred during a band of years or a single year. Referring to Table 1, an experience band can be taken as 1967 only or 1961-1967 or any other combination.

Assuming July first to be a common date for all placements, retirement rates for different age intervals can be computed by dividing the retirements during the age interval by the number of units of property surviving at the beginning of the age interval. For example, if the experience band for the account shown in Table 1 is decided to be 1965-1967, the retirement rate for the age interval 0-1/2 will equal (1/34) or 0.029. The survival rate would be 0.971 or (1-retirement rate). This survival rate of 0.971 is multiplied by the percent surviving at age zero (=100) to give the percent surviving at age 1/2 of 97.1%. Table 2 shows such calculations for the account shown in Table 1. An original survivor curve which shows the property surviving in service at successive ages can then be drawn by plotting the percent surviving against the age.

In practice, the retirement rate method frequently results in stub survivor curves, that is, curves that end at a percent surviving greater than zero. Further, many original data survivor curves, complete or stub, are irregular in shape to the extent that they exhibit sharp angles or horizontal segments. The stub curve must be extended to zero percent surviving and the irregular curve should be smoothed before the average service life is computed (1).

Table 2. Calculations for retirement rates and life table for account shown in Table 1 by the retirement rate method

Placement band: 1961 through 1967. Experience band: 1965 through 1967

Age interval	Number exposed to retirement at beginning of age interval	Number retired during age interval	Retirement rate for age interval	Survival rate for age interval	Percent surviving at beginning of age interval
1	2	3	4	5	6
0 - ½	34	1	0.029	0.971	100.0
½ - 1½	41	5	0.122	0.878	97.1
1½ - 2½	33	8	0.242	.757	85.2
2½ - 3½	28	9	.321	.679	64.6
3½ - 4½	11	4	.364	.636	43.9
4½					27.9

The smoothing and extension of original survivor curves can be done by judgment, statistical curve fitting and/or by matching to type or standard curves. The three methods were used in this study. Statistical curve fitting and matching to Iowa type curves were used because it was shown in an earlier study that ". . . no consistent superiority was enjoyed by either the Iowa type curve method or the use of orthogonal polynomials in estimating mortality dispersion" (35). Moreover, Cowles warned about depending solely on computer results of statistical analyses and urged the use of informed judgment based upon documented evidence (36).

Statistical curve fitting The analysis was conducted by fitting retirement ratios with polynomials from the first to third degree. A computer program (SELEC) developed by Northern States Power Company was used for this purpose (37). The program fits a curve of retirement ratios with polynomials and then compares the smoothed survivor curve resulting from this polynomial to the Iowa type curves to determine the best fit in a least squares sense. The dispersion and average service life best fitting the data are the primary output. Also included, however, are the total exposures for each age during the interval, and smoothed and actual retirement ratios and life tables.

Matching to Iowa type curves This was achieved by matching visually the original survivor curves, usually stubbed and irregular, to sets of different families of Iowa type curves drawn for various average service lives. In each case the best fitting Iowa type curve would determine the probable average service life and the retirement dispersion directly.

Judgment After performing statistical curve fitting and visual matching to Iowa type curves for all actuarial data obtained, judgment was exercised as to the reasonableness of the results especially by comparing the results from the two methods.

Simulated plant-record method (SPR)

This method encompasses both the "simulated plant balances method" introduced by Bauhan (38) and the "Simulated Plant Period Retirements Method" suggested by Garland (39).

The SPR Balances method is a trial and error procedure that attempts to duplicate the annual balances (or cumulative retirements) of a plant account by distributing the actual annual gross additions over time according to an assumed mortality distribution. The SPR Period Retirements method is also a trial and error procedure that first derives an average service life for an assumed mortality distribution that will provide a total volume of retirements over a test period of years equal in magnitude to the recorded volume of retirements over the same period of time. A computer program implementing both methods was developed by White and Cowles (40) in which the assumed or reference mortality distribution is the Iowa type distribution. This program was used for analyzing semi-actuarial data under study.

Service Patterns

Patterns of services or operation returns derivable from machinery and equipment may be ascertained by studying several indicators. Some of the indicators felt to be important for establishing trends are:

Operating costs including maintenance expenditures

Quality or adequacy of the service

Quantity produced

Hours used

Down-time

Scrap or spoilage

An attempt was made to collect data pertaining to such indicators. The data sought would give values, e.g. maintenance expenditures, according to age of the machinery and equipment category. Limited amount of data was collected for maintenance expenditures while data for other indicators could not be collected. Reasons given by the participating parties for being unable to supply such data were either that the data are not available or that they are too expensive and time consuming to prepare.

Regarding the maintenance expenditures data, it was felt that they should be adjusted for price level changes before analysis. This was done by indexing all data to 1967 as a base year by using the consumer price index.

EXPERIMENTAL DATA RESULTS

Two sorts of data were collected, data fit for life analyses and maintenance expenditures data. Sixteen companies supplied data of 19 machinery and equipment accounts for life analyses. Of these 16 companies, four supplied data of maintenance expenditures of their machinery and equipment accounts.

It should be pointed out that the sixteen industrial companies furnishing the data belong to the following areas of activities:

Cement manufacturing

Household appliance manufacturing

Machinery manufacturing

Printing

Meat processing

Grain processing

Primary metals processing

Life Analysis Data

Participating companies were asked to supply actuarial data if at all possible or semi-actuarial data if actuarial data could not be obtained. Of the nineteen machinery and equipment accounts supplied, thirteen contained actuarial information and six contained semi-actuarial information.

Actuarial data

The data were analyzed by matching original survivor curves to Iowa type curves and by statistical fitting of retirement ratios to polynomials of the first to third degree. Judgment was then exercised as to the results of these two methods. Table 3a shows the life tables and Table 3b shows summary of the results. Accounts were given code numbers as requested by the participating companies.

The experience band for applying the retirement rate method to actuarial data was normally chosen as the most recent five years.

Figures 23 and 24 show sample original and smoothed survivor curves drawn from actual data. While the original survivor curve in Figure 23 is relatively smooth, it extends only to about 70% surviving. On the other hand, the original survivor curve in Figure 24 is rather irregular but it extends to about 37% surviving. These two samples of survivor curves show some of the difficulties encountered in the analyses where desirable features, like curve smoothness and extension to low surviving percentages are not always encountered.

Semi-actuarial data

Six accounts which contained semi-actuarial information were analyzed using the simulated plant balances method. Table 4 shows the result of such analyses and Table 5 shows a sample SPR computer output.

Two criteria were used in selecting retirement dispersion and average service life from the SPR program output. The

Table 3a. Observed life tables, percent, for 13 machinery and equipment accounts with actuarial data

Age, years	Accounts 1	2	3	4	5	6	7	8	9	10	11	12	13
0.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.5	99.99	100.00	100.00	99.94	100.00	100.00	100.00	100.00	99.73	100.00	100.00	99.69	100.00
1.5	99.89	99.92	99.81	99.93	100.00	99.97	99.95	98.87	99.46	99.92	99.77	99.06	98.01
2.5	99.00	99.33	99.75	99.93	100.00	98.64	99.79	98.87	99.07	99.48	99.35	88.54	97.69
3.5	98.19	97.63	99.63	99.93	99.94	98.36	99.47	98.28	98.79	99.39	98.81	88.81	97.69
4.5	97.24	97.05	99.42	99.88	99.42	97.58	99.32	98.14	98.56	99.16	98.18	81.16	69.62
5.5	96.24	95.97	99.01	99.75	99.13	96.97	98.98	97.46	98.33	98.82	97.40	80.07	69.43
6.5	95.34	92.55	98.60	99.59	98.36	96.02	98.92	96.26	98.02	97.27	96.59	77.76	69.02
7.5	92.86	90.50	98.15	99.15	97.23	94.37	95.42	93.71	95.23	97.21	95.17	75.09	67.70
8.5	89.85	85.37	97.60	97.77	96.16	91.36	95.25	93.62	94.95	96.81	93.39	74.08	65.10
9.5	84.03	85.00	96.84	96.93	93.30	88.98	87.06	89.98	94.25	95.89	90.83	72.81	64.63
10.5	80.02	84.46	95.71	95.41	91.16	85.86	86.88	89.68	93.56	95.89	87.86		49.08
11.5	77.73	82.60	94.38	94.88	87.26	80.53	79.25	86.93	93.09	95.87	84.91		48.53
12.5	76.80	79.64	92.65	93.38	86.14	79.13	79.14	80.76	92.83	95.68	82.05		48.53
13.5	74.93	77.06	91.59	92.27	82.76	74.97	74.45	79.15	92.32	92.97	79.78		48.21
14.5	73.78	73.01	90.06	91.67	82.05	65.62	74.43	76.00	91.96	92.74	77.64		45.80
15.5	72.12	71.49	88.40	90.05	81.67	63.53	74.02	73.65	91.46	92.48	74.79		45.35
16.5	70.29	69.95	86.90	88.97	81.67	61.62	74.02	72.47	90.47	92.09	72.59		43.32
17.5	69.10	69.03	86.18	88.46	78.26	60.52	73.29	70.73	89.61	91.47	69.51		43.19
18.5	67.02	66.02	85.76	87.24	77.83	56.37	73.29	69.78	88.79	91.36	67.19		37.37
19.5	62.44	62.79	85.17	85.13	74.37		73.29	67.74	86.90	90.35	64.77		37.37
20.5	55.82	59.22	82.86	83.83	72.49		73.29	65.49	86.46	90.20			
21.5	54.42	59.22	80.81	82.09	69.09		73.29	64.50	85.63	90.06			
22.5	53.37	58.77	79.38	79.54	66.74		64.66	64.01	84.16	89.60			
23.5	50.59	56.60	78.75	78.34	66.25		64.52	60.93	83.77	87.70			
24.5	49.22	56.12	77.64	74.91	65.19		63.01	60.45	83.43				
25.5	48.37	51.45	77.38	72.51	64.31		61.14	57.83	82.12				

26.5	57.19	57.31	81.31
27.5	57.19	56.92	79.62
28.5	57.19	55.08	79.37
29.5	57.19		78.63
30.5	57.19		77.57
31.5	38.57		77.31
32.5	13.00		76.95
33.5	13.00		76.42
34.5	13.00		76.24
35.5			76.18
36.5			75.84
37.5			75.69
38.5			75.59
39.5			75.25
40.5			75.25
41.5			75.14
42.5			75.00
43.5			75.00
44.5			74.91
45.5			74.91
46.5			74.68
47.5			73.49
48.5			73.49
49.5			73.49
50.5			43.28

Table 3b. Summary of analyses of actuarial data for thirteen machinery and equipment accounts

Account number	Iowa retirement dispersion and average service life in years estimated by: Matching to Iowa type curves	Fitting of retirement ratios to polynomials	Dispersion and average service life chosen
1	L_1-23	L_0-31	L_1-25
2	L_0-27	L_0-29.1	L_0-29
3	L_1-40	L_1-42.4	L_1-40
4	S_1-32	$L_{1.5}$-39	S_1-32
5	L_1-32	SC-36.5	L_1-32
6	L_2-18	$L_{1.5}$-22.1	L_2-18
7	S_2-27	$S_{0.5}$-27	S_2-27
8	L_1-29	$L_{0.5}$-31	L_1-30
9	L_1-42	R_2-48	L_1-42
10	R_2-42	R_3-38.6	R_2-42
11	S_0-25	$L_{1.5}$-26.1	S_0-25
12	R_2-12	R_2-10.2	R_2-12
13	L_1-17	O_2-13.5	L_1-17

N.B. Iowa type curves designated by half-number subscripts are intermediate curves, e.g. $L_{0.5}$ is an intermediate curve between L_0 and L_1. The SC curve is the same as the O_1 curve.

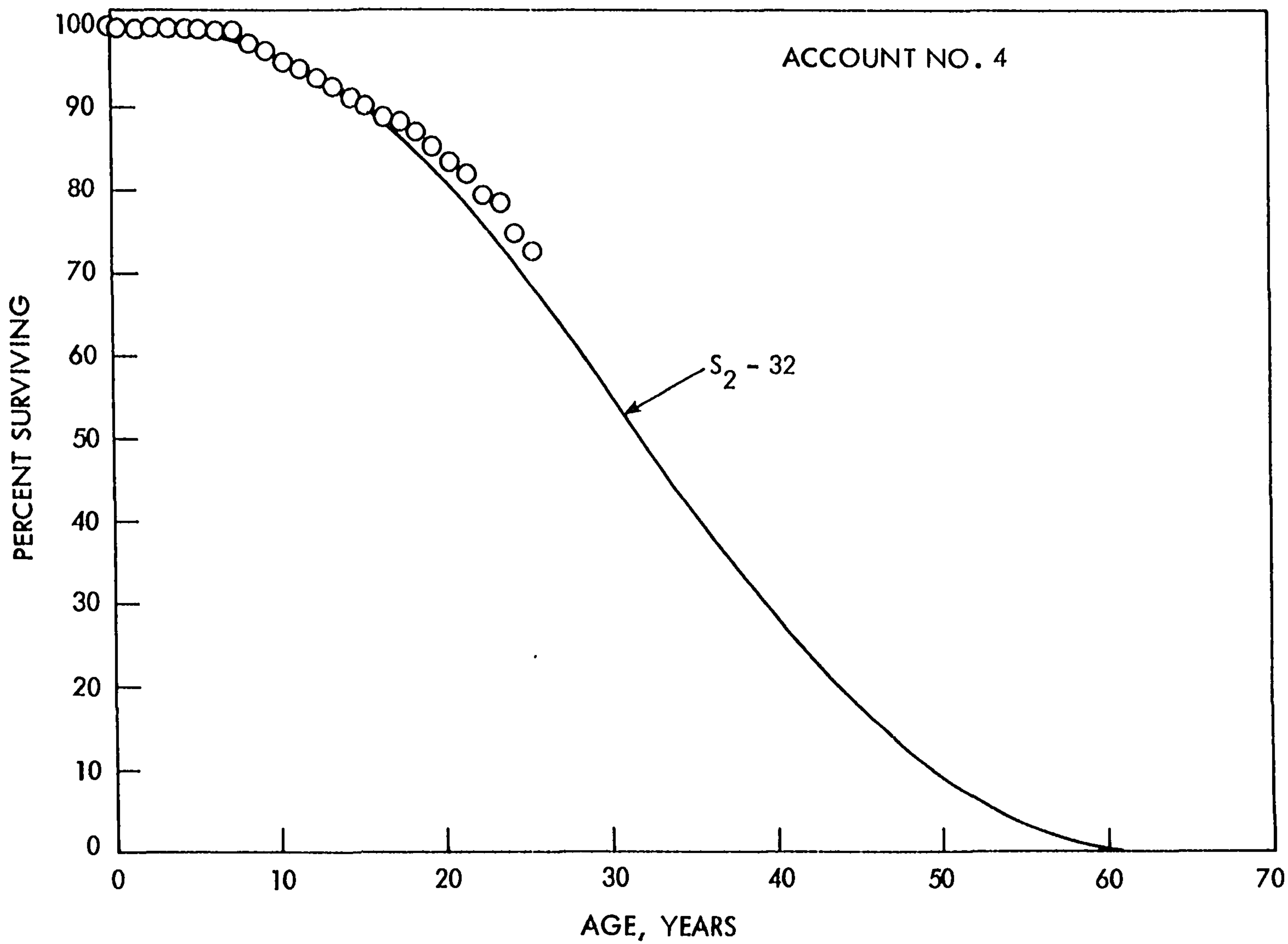

Fig. 23. Sample original and smoothed survivor curves

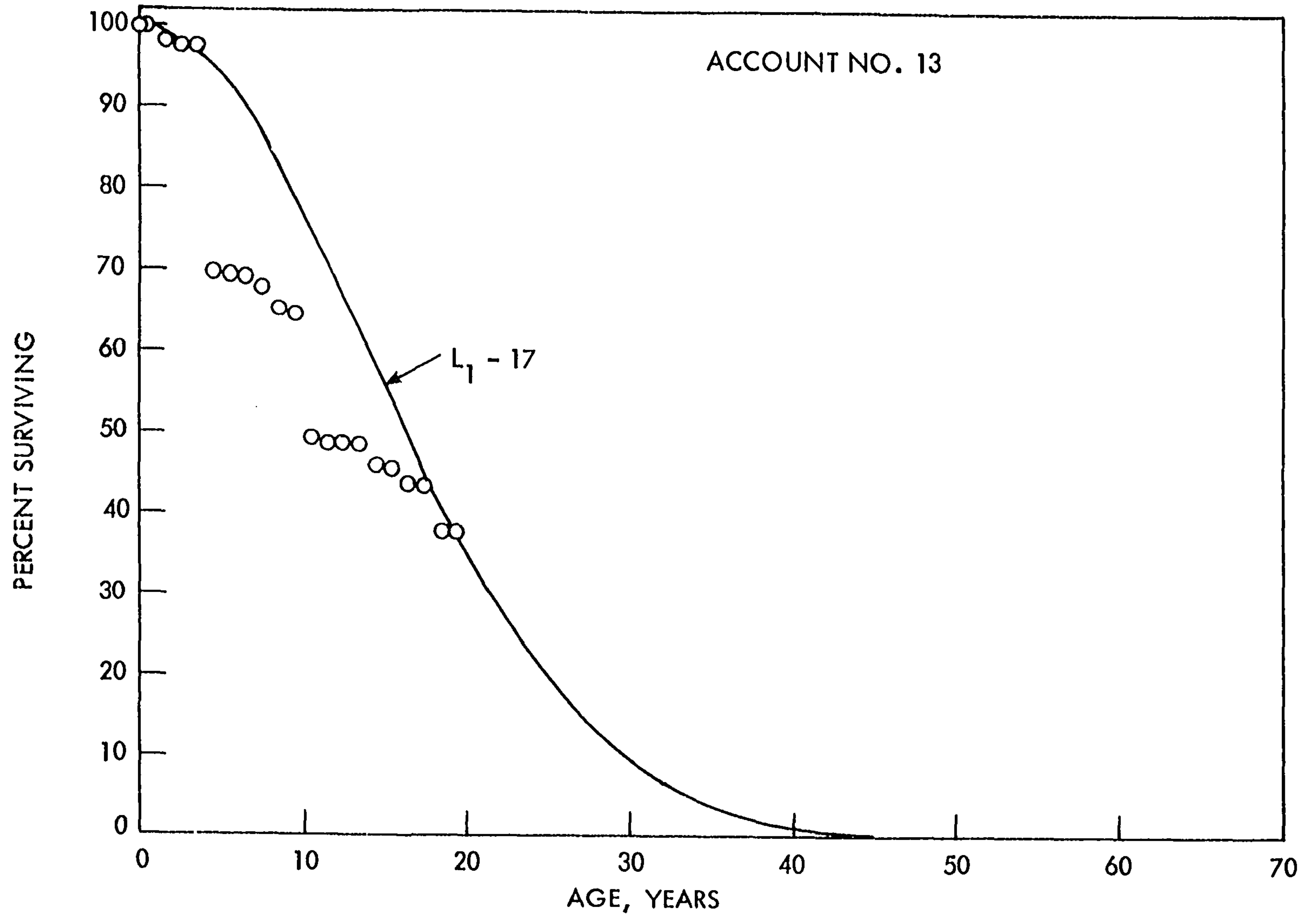

Fig. 24. Sample original and smoothed survivor curves

Table 4. Summary of analyses of semi-actuarial data for six machinery and equipment accounts

Account number	Iowa retirement dispersion and estimated average service life in years
14	R_3 - 31.3
15	L_3 - 25.9
16	R_3 - 17.0
17	S_5 - 18.4
18	R_3 - 22.4
19	S_6 - 19.5

first is an index of variation defined as (40):

Index of variation =

$$1000 \left[\frac{\text{Sum of squared differences from actual balances}}{\text{Number of test years}} \right]^{1/2} \Big/ \text{Average actual balance}$$

The smaller the index of variation the better would be the fit of the simulated balances to actual balances.

The second criterion is the retirement experience index which is "the portion of the oldest addition which would have been retired as of the date of the study had the property

Table 5. Sample SPR output (no. of test points = 5; interval between test points = 0; last test point = 1974)

Dispersion	Average service life (years)	Sum of squares diff.	Index of variation	Ret. exp. index
R2.5	33.3	0.1514E 04	8	69.62
R3	31.3	0.1529E 04	8	82.95
R2	35.8	0.1547E 04	8	57.07
S0.5	38.1	0.1571E 04	8	52.99
S1	34.8	0.1573E 04	8	62.26
S2	31.2	0.1577E 04	8	78.95
S1.5	32.7	0.1580E 04	8	71.00
R1.5	40.4	0.1581E 04	8	44.64
L2	35.1	0.1585E 04	8	65.79
S0	42.6	0.1589E 04	8	45.01
L1.5	38.4	0.1592E 04	8	57.19
L1	42.5	0.1605E 04	8	49.55
L0.5	49.4	0.1607E 04	8	41.98
L0	58.4	0.1615E 04	8	36.21
R1	46.5	0.1617E 04	8	36.54
S6	26.6	0.1625E 04	8	100.00
S-.5	53.4	0.1628E 04	8	34.61
S3	29.5	0.1631E 04	8	91.28
R0.5	57.2	0.1646E 04	8	30.71
D2	78.6	0.1659E 04	9	28.24
SC	70.1	0.1660E 04	9	28.18
L3	31.1	0.1662E 04	9	80.18
D3	114.5	0.1664E 04	9	27.49
D4	158.6	0.1665E 04	9	27.27
L4	28.9	0.1737E 04	9	92.52
R4	29.5	0.1836E 04	9	97.59
L5	27.6	0.1842E 04	9	98.74
S4	28.2	0.1920E 04	9	99.08
S5	27.0	0.2027E 04	9	100.00
R5	27.4	0.2195E 04	10	100.00
SQ	28.9	0.1150E 05	23	100.00

experienced the indicated dispersion and average service life. A value at or near 100% suggests at least a complete life cycle of the experience considered in the analysis" (40). If the retirement experience index was low, e.g. 50%, the particular fit was discarded even though the index of variation was small. This was done because in that case there has not been enough experience with the account for it to exhibit a conclusive life characteristic.

Results

Based on the limited sample of 19 machinery and equipment accounts, the Iowa type curves were found to be appropriate for analyzing both the actuarial and the semi-actuarial data.

It was also found that there is no evidence that machinery and equipment in similar industries have similar retirement dispersions or similar probable average service lives.

In general, the probable average service lives estimated for the machinery and equipment fell between 12 and 40 years. It is felt that this range can be divided to three subranges of reasonable average service lives, viz.:

Range I	10-20 years
Range II	20-30 years
Range III	30-40 years

Maintenance Expenditures Data

Data relating maintenance expenditures to age of industrial machinery and equipment were collected from four participating companies. After adjusting for changing price levels by the consumer price index, regression analyses were performed on the data to establish general trends.

Table 6 shows the adjusted maintenance expenditures by age as percents of original investments for the four companies.

The data for companies A, B and C fitted well to positive sloping straight lines while fits to higher order polynomials proved to be insignificant. Company's D data best fitted a third degree polynomial which show slight trend of decreasing expenditures with age. However, this may be due to putting machines representing significant investment on stand-by as observed by plant inspection.

Figures 25 through 28 show the actual data and regressed polynomials for the four companies. It is not claimed that the above sample can lead to definite conclusions but it agrees in a general way with Terborgh's empirical findings that, in general, the repair costs of machinery and equipment increases with age (16).

Indexing the data proved to be useful in assessing the results conservatively. In periods of rising price levels, i.e. inflation, indexing would show slower increase of maintenance expenditures with age. As an illustration, Figure 29 is

Table 6. Indexed maintenance expenditures by age for four companies

Average age	Indexed maintenance expenditures as percents of original investment			
	Company A	Company B	Company C	Company D
0.25	1.70	1.10	1.66	2.21
1	3.94	4.70	2.72	2.88
2	4.89	2.38	2.11	4.06
3	2.89	2.99	2.10	7.55
4	3.38	3.00	2.12	7.84
5	3.47	3.18	2.28	8.44
6	3.85	4.04	1.88	8.94
7	6.13	5.09	2.02	6.93
8	7.12	4.54	3.36	4.79
9	6.83	5.29	3.28	5.48
10	5.78	4.31	4.85	3.63
11	4.15	4.48	4.72	2.95
12	5.63	4.85	4.65	3.01
13	7.20	6.38	5.09	3.77
14	5.69	5.27	4.23	3.28
15	13.41	6.18	4.84	3.35
16	4.12	3.02	5.07	3.83
17	5.54	0.78	3.15	3.87
17	5.82	1.05	3.50	4.42
19	5.96	2.06	4.80	3.27
20		2.16	4.44	2.71
21		14.17	2.98	4.08
22		13.76	7.11	4.04
23		6.38	6.11	4.18
24		10.63	7.42	5.42
25		8.36	2.72	4.75
26		10.48		4.07
27		3.21		

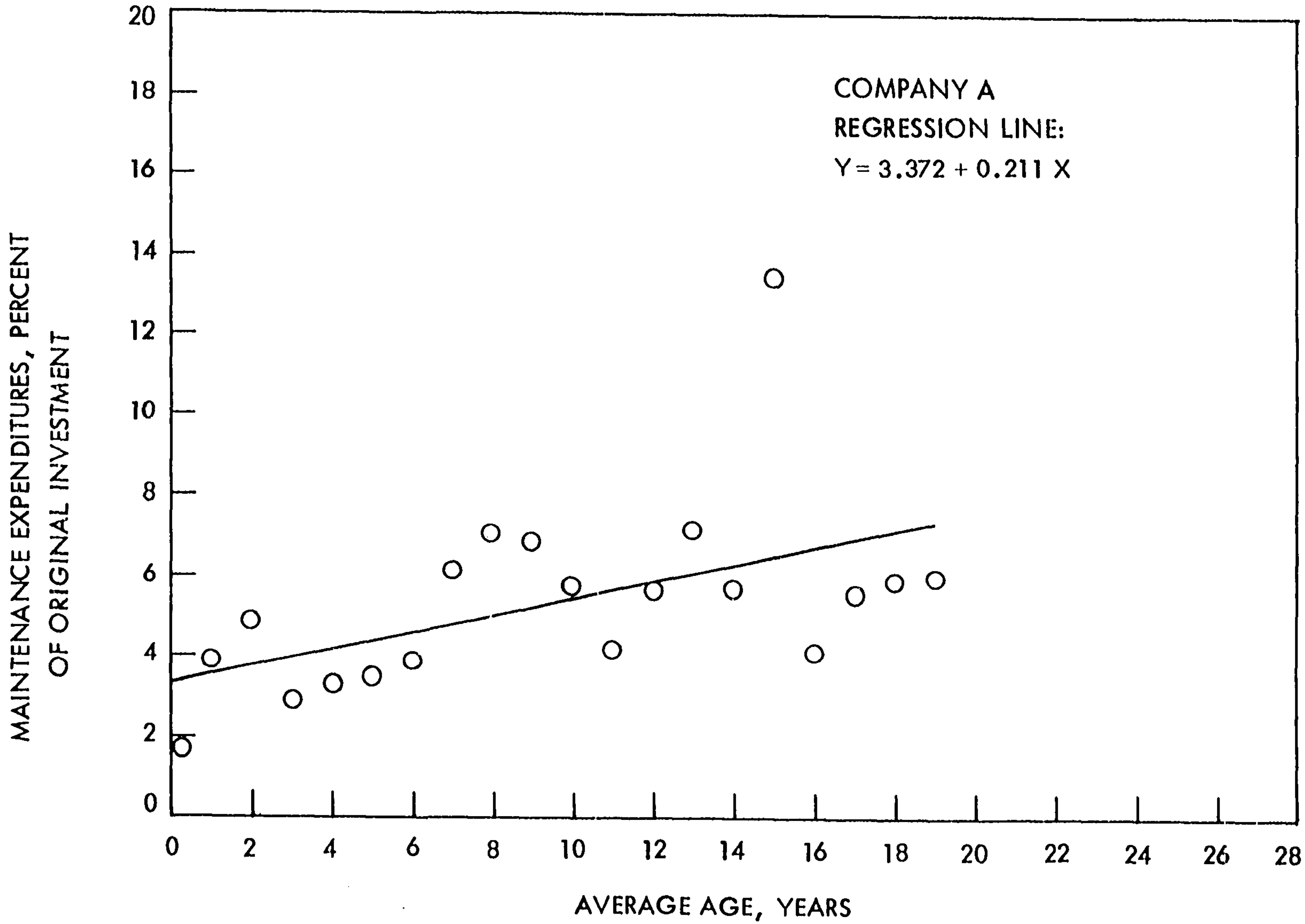

Fig. 25. Relation between age and maintenance expenditures

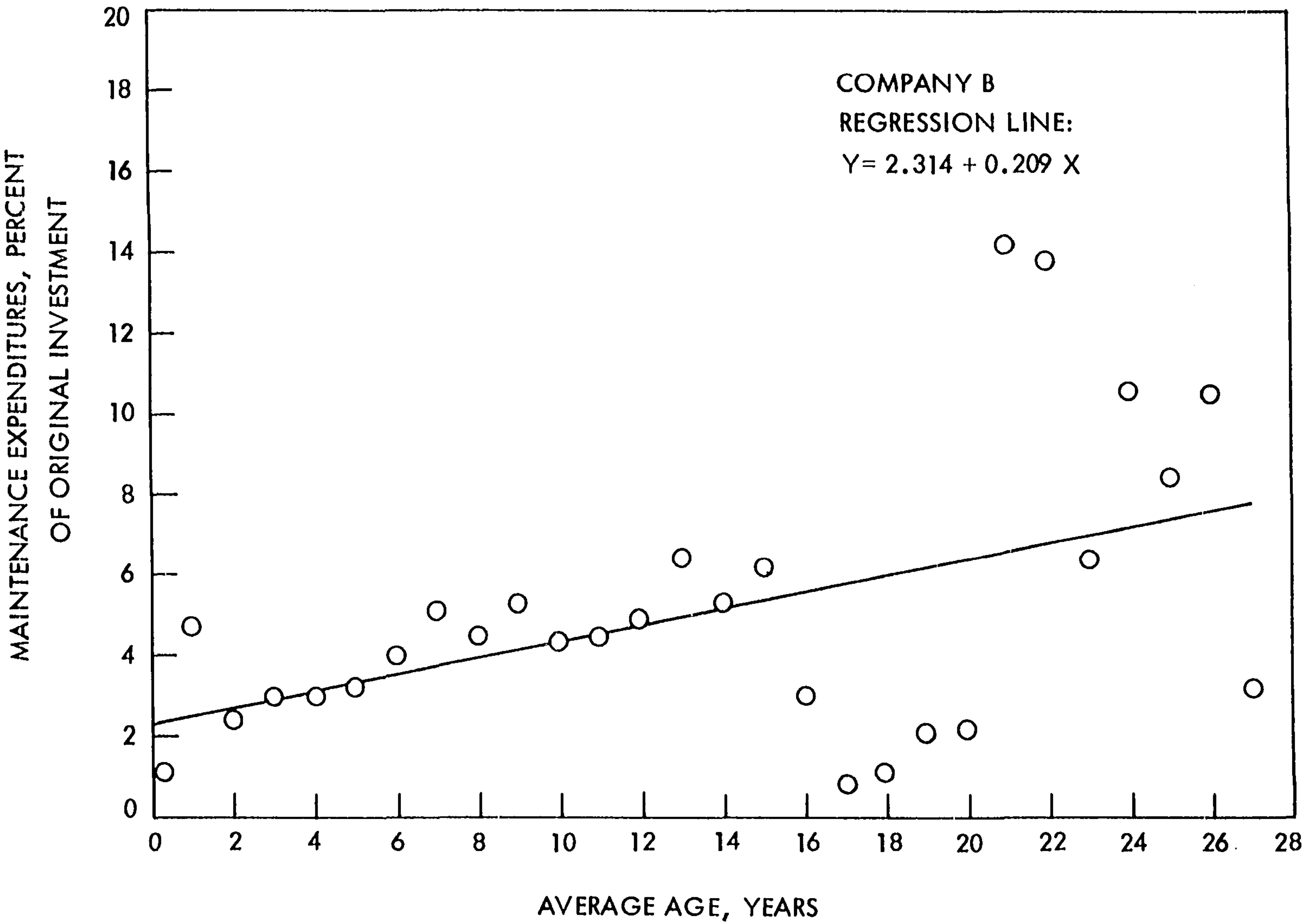

Fig. 26. Relation between age and maintenance expenditures

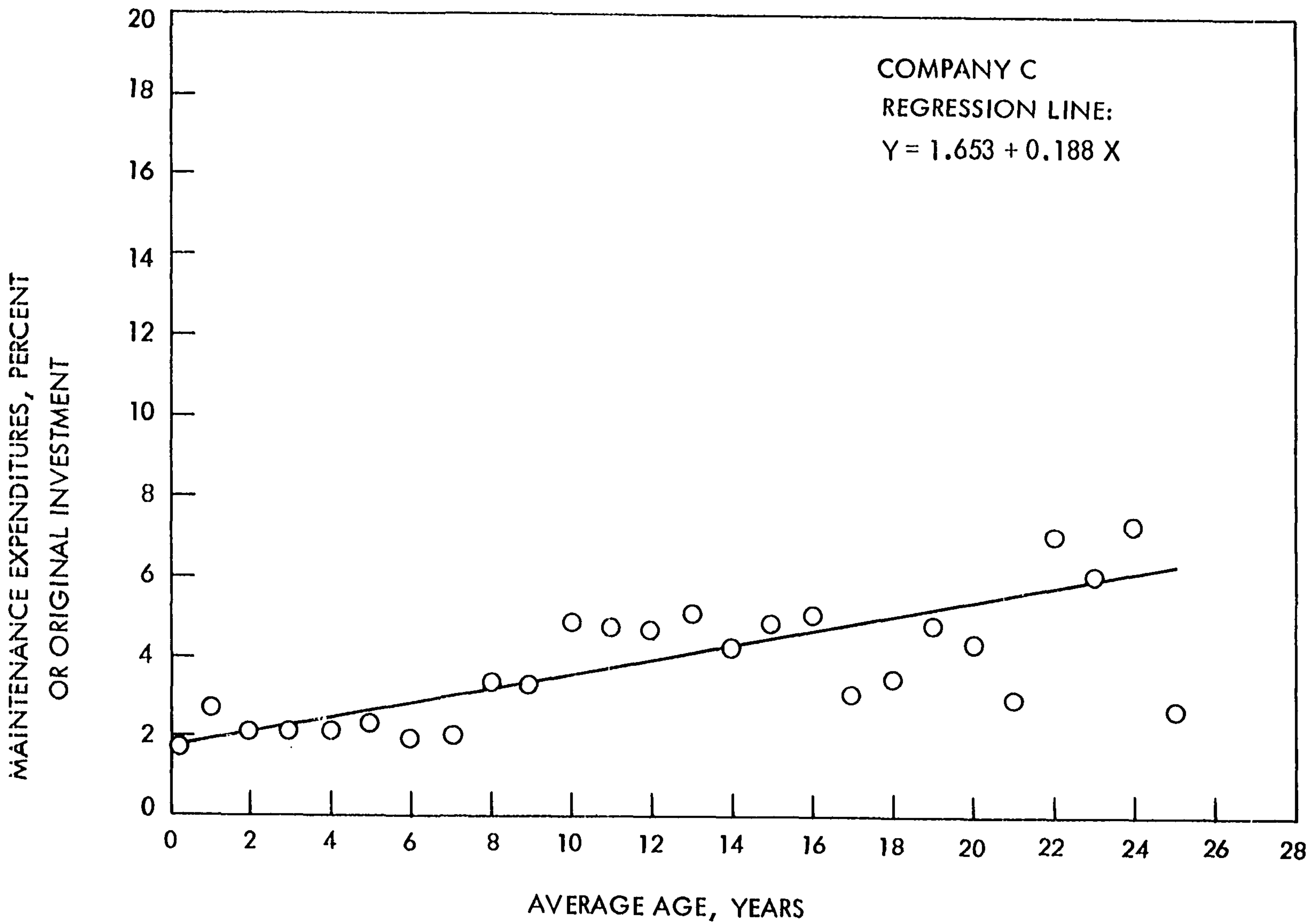

Fig. 27. Relation between age and maintenance expenditures

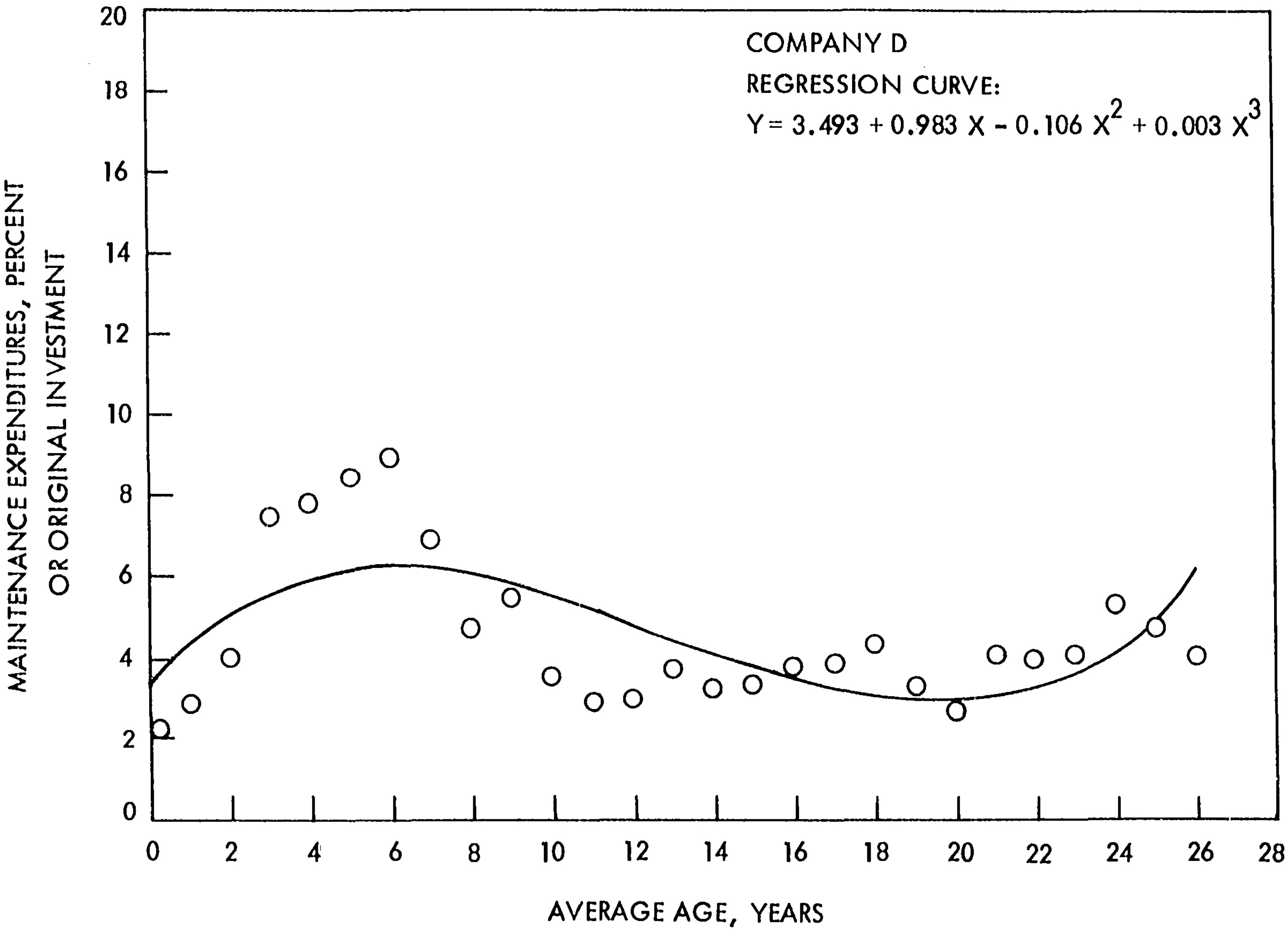

Fig. 28. Relation between age and maintenance expenditures

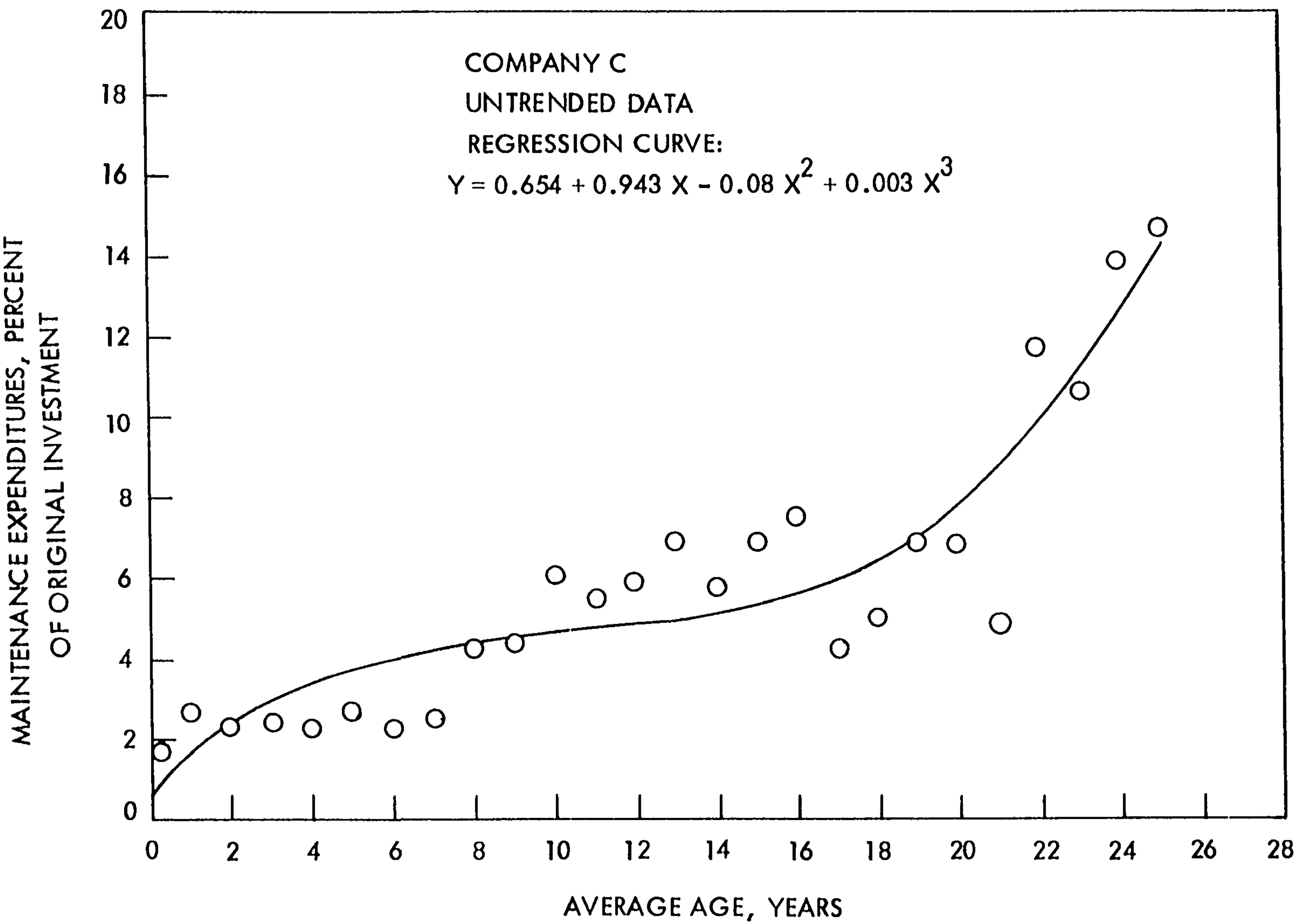

Fig. 29. Relation between age and maintenance expenditures

presented which shows maintenance expenditures data without indexing for company C. Comparing Figures 26 and 29, the effect of indexing maintenance expenditures to base year 1967 is apparent.

MASS ACCOUNTS, PRICE LEVELS AND DISCOUNT RATES

Several examples were shown of the application of the model to units and groups of units of property. Now, application to mass accounts will be discussed.

Mass Accounts

Accounts composed of several vintage groups, i.e., mass accounts, are treated by first identifying or estimating the survivors of each of the constituent vintage groups according to age. Second, the value of each group is computed separately according to its age. Third, the corresponding value of the mass account is then arrived at by summing the values of the constituent groups.

As an example, suppose a mass account is thought to follow R_2 Iowa type retirement dispersion and estimated to have 10 years probable average service life. The account has 1,000,000 surviving dollars. These surviving dollars belong to several vintage groups which are identified by age and the corresponding surviving dollars in each vintage group. The value of the mass account can then be arrived at by summing the values of the constituent vintage groups. Assuming an effective annual discount rate, r, of 10%, progression rate, T, of 1.3 and zero salvage ratio, the values of these vintage groups are first computed by using Equation 12. Table 7 illustrates the

Table 7. Estimating the value of a mass account. R_2-10 Iowa type. T = 1.3. r = 10%. S = 0

Age of vintage groups, years	Surviving dollars	Modified condition percent factor	Estimated value of vintage group[a]
0	0	1.0000	0
1	50,000	0.8737	43685
2	100,000	0.7654	76540
3	60,000	0.6671	40026
4	130,000	0.5766	74958
5	0	0.4935	0
6	50,000	0.4176	20880
7	50,000	0.3493	17465
8	30,000	0.2886	8658
9	70,000	0.2358	16506
10	120,000	0.1909	22908
11	0	0.1532	0
12	40,000	0.1221	4884
13	90,000	0.0961	8649
14	50,000	0.0740	3700
15	60,000	0.0550	3300
16	40,000	0.0393	1572
17	50,000	0.0297	1485
18	10,000	0.0287	287
19	0	0.0000	0
	$1,000,000		$345,503 Total = value of mass account

[a]Col. 4 = (col. 2)(col. 3).

procedure in which it is shown that the value of the mass account came out to be \$345,503.

Price Levels and Discount Rates

The question now arises as to what rate of return to use in the proposed model. Will it be adjusted to reflect price level changes or will it be "inflation or deflation free" rate of return?

The price level adjusted rate of return, y, is related to the constant-purchasing power unadjusted rate of return, r, and the rate of inflation or deflation, I, as follows (41):

$$y = (1+r)(1+I) - 1 \tag{53}$$

For estimating the value of the property by discounting future operation returns and salvage of the property, it makes no difference whether the constant-purchasing power rate of return is used to discount future operation returns in constant-purchasing power dollars or the adjusted rate of return is used to discount future operation returns adjusted for price level changes.

For application of the model presented here, it is recommended that the unadjusted constant-purchasing power rate of return be used. This is because the actual operation returns can not be estimated and "calculated" operation returns are estimated instead depending on the pattern of future services. These "calculated" operation returns will be in constant-purchasing power dollars.

This estimate of value can then be adjusted for price level changes, whether due to inflation or deflation, by applying an appropriate price index, e.g., the wholesale price index or the implicit price deflators for the time span between age zero and the age at valuation date.

CONCLUSIONS

Several conclusions can be drawn from this research effort. They can be enumerated as:

1. The need for developing a general valuation model of industrial property units or groups whose services are uniform or declining with age was demonstrated. Accordingly a model was proposed and developed for computing both the modified condition percent factor and the estimated value at every half year of the property life. The model presents the analyst with a continuous spectrum of service patterns from which he would select a typical pattern appropriate for the property under study and its use.

2. The proposed valuation model was shown to be suitable for application to property units, vintage groups and mass accounts composed of several vintage groups.

3. The progression rate, T, of the services or operation returns has a significant effect on the modified condition percent factors and property values estimated.

4. Life analysis data of machinery and equipment, both actuarial and semi-actuarial, collected from 16 companies representing various Iowa industries were analyzed. It was found that the well-known Iowa type survivor curves are appropriate for analyzing the data and making life forecasts.

5. It was found that there is no evidence, from sample data analyzed, that machinery and equipment in similar industries have similar mortality characteristics, i.e., similar retirement dispersions or similar probable average service lives.

6. Probable average service lives estimated for groups of machinery and equipment studied fell between 12 and 40 years.

Further research is recommended for the study of patterns of services or operation returns derivable from machinery and equipment in various industries. This would include the study of service indicators such as quantity and quality of service, operating costs and their derivatives, time used, down time and scrap or spoilage.

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APPENDIX

Table A-1. Operation return ratios for a property unit. Probable life = 10 years, S = 0

Age, years	T=0.7		T=0.9		T=1.0	
	r=6%	r=10%	r=6%	r=10%	r=6%	r=10%
0.5	0.3307	0.3499	0.1633	0.1802	0.1168	0.1317
1.0	0.2314	0.2448	0.1447	0.1597	0.1110	0.1251
1.5	0.1619	0.1713	0.1280	0.1412	0.1051	0.1185
2.0	0.1133	0.1198	0.1129	0.1246	0.0993	0.1120
2.5	0.0792	0.0838	0.0994	0.1097	0.0934	0.1054
3.0	0.0554	0.0586	0.0872	0.0962	0.0876	0.0988
3.5	0.0387	0.0409	0.0762	0.0841	0.0818	0.0922
4.0	0.0270	0.0286	0.0663	0.0732	0.0759	0.0856
4.5	0.0188	0.0199	0.0574	0.0634	0.0701	0.0790
5.0	0.0131	0.0138	0.0494	0.0545	0.0642	0.0724
5.5	0.0091	0.0096	0.0422	0.0466	0.0584	0.0659
6.0	0.0063	0.0066	0.0357	0.0394	0.0526	0.0593
6.5	0.0043	0.0046	0.0299	0.0330	0.0467	0.0527
7.0	0.0029	0.0031	0.0247	0.0272	0.0409	0.0461
7.5	0.0020	0.0021	0.0199	0.0220	0.0350	0.0395
8.0	0.0013	0.0014	0.0157	0.0173	0.0292	0.0329
8.5	0.0008	0.0009	0.0118	0.0131	0.0234	0.0263
9.0	0.0005	0.0005	0.0084	0.0093	0.0175	0.0198
9.5	0.0003	0.0003	0.0053	0.0059	0.0117	0.0132
10.0	0.0001	0.0003	0.0025	0.0028	0.0058	0.0066

T=1.1		T=1.3		T=∞	
r=6%	r=10%	r=6%	r=10%	r=6%	r=10%
0.0933	0.1069	0.0772	0.0898	0.0669	0.0794
0.0917	0.1051	0.0770	0.0897	0.0669	0.0794
0.0899	0.1030	0.0769	0.0895	0.0669	0.0794
0.0879	0.1008	0.0767	0.0892	0.0669	0.0794
0.0858	0.0983	0.0764	0.0889	0.0669	0.0794
0.0834	0.0955	0.0760	0.0885	0.0669	0.0794
0.0808	0.0925	0.0756	0.0880	0.0669	0.0794
0.0779	0.0892	0.0750	0.0873	0.0669	0.0794
0.0747	0.0856	0.0742	0.0864	0.0669	0.0794
0.0712	0.0816	0.0732	0.0852	0.0669	0.0794
0.0674	0.0772	0.0719	0.0837	0.0669	0.0794
0.0631	0.0723	0.0702	0.0818	0.0669	0.0794
0.0585	0.0670	0.0681	0.0792	0.0669	0.0794
0.0534	0.0611	0.0652	0.0759	0.0669	0.0794
0.0477	0.0547	0.0615	0.0716	0.0669	0.0794
0.0416	0.0476	0.0567	0.0660	0.0669	0.0794
0.0348	0.0398	0.0504	0.0587	0.0669	0.0794
0.0273	0.0312	0.0423	0.0492	0.0669	0.0794
0.0190	0.0218	0.0317	0.0369	0.0669	0.0794
0.0100	0.0114	0.0179	0.0208	0.0669	0.0794

Table A-2. Operation return ratios for a property unit. Probable life = 10 years, T = 1.3 and r = 6%

Age, years	S = 0.0	S = 0.1	S = 0.2	S = 0.3	S = 0.4	S = 0.5
0.5	0.0772	0.0728	0.0685	0.0642	0.0599	0.0556
1.0	0.0770	0.0727	0.0684	0.0641	0.0598	0.0555
1.5	0.0769	0.0726	0.0683	0.0640	0.0597	0.0554
2.0	0.0767	0.0724	0.0681	0.0638	0.0595	0.0553
2.5	0.0764	0.0721	0.0679	0.0636	0.0593	0.0551
3.0	0.0760	0.0718	0.0676	0.0633	0.0591	0.0548
3.5	0.0756	0.0714	0.0671	0.0629	0.0587	0.0545
4.0	0.0750	0.0708	0.0666	0.0624	0.0582	0.0541
4.5	0.0742	0.0701	0.0659	0.0618	0.0576	0.0535
5.0	0.0732	0.0691	0.0651	0.0610	0.0569	0.0528
5.5	0.0719	0.0679	0.0639	0.0599	0.0559	0.0518
6.0	0.0702	0.0663	0.0624	0.0585	0.0546	0.0506
6.5	0.0681	0.0643	0.0605	0.0566	0.0528	0.0490
7.0	0.0652	0.0616	0.0579	0.0543	0.0506	0.0470
7.5	0.0615	0.0581	0.0546	0.0512	0.0478	0.0443
8.0	0.0567	0.0535	0.0503	0.0472	0.0440	0.0408
8.5	0.0504	0.0476	0.0448	0.0420	0.0391	0.0363
9.0	0.0423	0.0399	0.0375	0.0352	0.0328	0.0305
9.5	0.0317	0.0299	0.0281	0.0264	0.0246	0.0228
10.0	0.0179	0.0169	0.0159	0.0149	0.0139	0.0129

Table A-3. Modified condition percents for a property unit with 10 years probable life for selected progression and discount rates

Age, years	T = 0.7		T = 0.9		T = 1.0	
	r=6%	r=10%	r=6%	r=10%	r=6%	r=10%
1	48.80	48.82	74.71	75.13	82.88	83.67
2	23.73	23.76	54.72	55.37	67.10	68.41
3	11.46	11.49	39.05	39.79	52.75	54.32
4	5.47	5.49	26.92	27.63	39.90	41.52
5	2.55	2.57	17.67	18.29	28.66	30.14
6	1.14	1.15	10.81	11.29	19.11	20.31
7	0.47	0.48	5.92	6.24	11.36	12.21
8	0.16	0.17	2.65	2.82	5.51	5.99
9	0.04	0.04	0.75	0.81	1.69	1.85
10	0.00	0.00	0.00	0.00	0.00	0.00

T = 1.1		T = 1.3		T = 2.0		T = ∞	
r=6%	r=10%	r=6%	r=10%	r=6%	r=10%	r=6%	r=10%
87.22	88.28	90.35	91.61	91.87	93.18	92.41	93.72
74.40	76.22	80.19	82.46	83.25	85.69	84.37	86.82
61.69	63.99	69.53	72.53	74.11	77.44	75.85	79.23
49.29	51.76	58.42	61.82	64.43	68.37	66.81	70.88
37.43	39.80	46.96	50.41	54.17	58.40	57.23	61.69
26.43	28.45	35.35	38.49	43.31	47.44	47.08	51.59
16.65	18.15	23.95	26.44	31.86	35.47	36.32	40.47
8.58	9.47	13.38	14.98	19.97	22.59	24.91	28.24
2.79	3.12	4.76	5.41	8.36	9.60	12.82	14.79
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A-4. Modified condition percents for property units. r = 6%

Age, % of probable life	T = 0.9 Probable life, years			T = 1.0 Probable life, years			T = 2.0 Probable life, years		
	10	20	30	10	20	30	10	20	30
10	74.71	64.03	52.84	82.88	83.87	84.80	91.87	94.15	95.82
20	54.72	40.53	27.79	67.10	68.71	70.26	83.25	87.57	90.85
30	39.05	25.22	14.50	52.75	54.63	56.53	74.11	80.19	84.92
40	26.92	15.29	7.45	39.90	41.77	43.74	64.43	71.89	77.87
50	17.67	8.91	3.73	28.66	30.27	32.09	54.17	62.56	69.46
60	10.81	4.88	1.78	19.11	20.31	21.78	43.31	52.08	59.46
70	5.92	2.40	0.78	11.36	12.08	13.08	31.86	40.31	47.53
80	2.65	0.96	0.28	5.51	5.78	6.30	19.97	27.09	33.34
90	0.75	0.24	0.06	1.69	1.67	1.79	8.36	12.49	16.49
100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A-5. Modified condition percents for property groups. S_2 Iowa type dispersion. Average service life = 10 years. r = 6%

Age, years	T=0.7	T=0.9	T=1.0	T=2.0	T=∞
1	48.01	72.82	80.85	90.19	91.03
2	22.83	52.09	64.12	80.13	81.76
3	10.75	36.68	50.00	70.24	72.55
4	5.02	25.50	38.48	60.92	63.77
5	2.33	17.57	29.34	52.44	55.69
6	1.08	12.03	22.23	44.92	48.46
7	0.49	8.20	16.77	38.36	42.09
8	0.23	5.57	12.61	32.69	36.52
9	0.10	3.77	9.46	27.80	31.68
10	0.05	2.54	7.06	23.58	27.46
11	0.02	1.70	5.25	19.94	23.79
12	0.01	1.13	3.87	16.78	20.56
13	0.00	0.74	2.83	14.02	17.73
14	0.00	0.48	2.05	11.62	15.23
15	0.00	0.31	1.46	9.54	13.03
16	0.00	0.20	1.03	7.76	11.11
17	0.00	0.12	0.73	6.31	9.51
18	0.00	0.08	0.56	5.38	8.45
19	0.00	0.00	0.00	0.00	0.00

Table A-6. Modified condition percents for property groups. S_2 Iowa type dispersion. Average service life = 10 years. Progression rate, T = 1.3

Age, years	r=0%	r=2%	r=6%	r=10%	r=14%
1	86.06	86.84	88.25	89.46	90.50
2	72.81	74.14	76.56	78.68	80.52
3	60.65	62.32	65.40	68.15	70.58
4	49.93	51.76	55.20	58.32	61.12
5	40.74	42.62	46.19	49.48	52.48
6	33.05	34.89	38.42	41.73	44.79
7	26.70	28.44	31.83	35.05	38.06
8	21.51	23.11	26.28	29.34	32.24
9	17.27	18.72	21.63	24.47	27.20
10	13.81	15.10	17.73	20.33	22.86
11	10.99	12.12	14.45	16.80	19.10
12	8.68	9.67	11.70	13.78	15.84
13	6.80	7.64	9.39	11.20	13.02
14	5.27	5.97	7.45	9.00	10.57
15	4.03	4.60	5.84	7.14	8.47
16	3.04	3.51	4.52	5.59	6.70
17	2.30	2.68	3.50	4.38	5.30
18	1.84	2.16	2.87	3.63	4.43
19	0.00	0.00	0.00	0.00	0.00

Table A-7. Modified condition percents for property groups. Average service life = 10 years. T = 1.3 and r = 6%

Age, years	S_0 type dispersion	S_2 type dispersion	S_4 type dispersion	S_6 type dispersion
1	85.43	88.25	89.81	90.25
2	74.44	76.56	79.13	79.98
3	65.49	65.40	68.02	69.23
4	57.91	55.20	56.61	58.04
5	51.31	46.19	45.25	46.55
6	45.46	38.42	34.61	34.97
7	40.20	31.83	25.53	23.74
8	35.40	26.28	18.50	13.88
9	31.00	21.63	13.43	7.58
10	26.92	17.73	9.90	5.00
11	23.12	14.45	7.47	4.14
12	19.58	11.70	5.79	3.78
13	16.27	9.39	4.63	0.00
14	13.19	7.45	3.83	
15	10.34	5.84	3.22	
16	7.76	4.52	0.00	
17	5.47	3.50		
18	3.61	2.87		
19	2.78	0.00		
20	0.00			

Table A-8. Modified condition percents for property groups. Average service life = 10 years. T = 1.3 and r = 6%

Age, years	L_2 type dispersion	S_2 type dispersion	R_2 type dispersion
1	86.75	88.25	87.37
2	74.57	76.56	76.54
3	63.57	65.40	66.71
4	53.98	55.20	57.66
5	46.23	46.19	49.35
6	40.24	38.42	41.76
7	35.65	31.83	34.93
8	32.02	26.28	28.86
9	29.00	21.63	23.58
10	26.33	17.73	19.09
11	23.86	14.45	15.32
12	21.51	11.70	12.21
13	19.29	9.39	9.61
14	17.19	7.45	7.40
15	15.25	5.84	5.50
16	13.47	4.52	3.93
17	11.83	3.50	2.97
18	10.33	2.87	2.87
19	8.96	0.00	0.00
20	7.69		
21	6.53		
22	5.46		
23	4.49		
24	3.62		
25	2.86		
26	2.36		
27	0.00		

Table A-9. Modified condition percents for property groups. S_2 Iowa type dispersion. r = 6%

Age, years	T = 0.9			T = 1.3		
	5 years average service life	10 years average service life	15 years average service life	5 years average service life	10 years average service life	15 years average service life
0.5	77.23	85.53	88.02	85.62	94.15	96.77
1.5	44.03	61.73	67.82	60.08	82.37	90.11
2.5	24.21	43.79	51.87	40.69	70.88	83.27
3.5	13.16	30.62	39.37	27.27	60.16	76.40
4.5	7.18	21.19	29.69	18.31	50.54	69.63
5.5	3.94	14.55	22.26	12.38	42.15	63.10
6.5	2.19	9.94	16.61	8.48	34.98	56.92
7.5	1.27	6.76	12.34	6.04	28.93	51.15
8.5	0.87	4.59	9.14	5.00	23.85	45.82
9.5	0.00	3.10	6.75	0.00	19.59	40.95
10.5		2.08	4.97		16.02	36.52
11.5		1.39	3.65		13.02	32.51
12.5		0.92	2.68		10.50	28.88
13.5		0.60	1.96		8.38	25.60
14.5		0.39	1.43		6.61	22.64
15.5		0.25	1.04		5.14	19.96
16.5		0.16	0.75		3.97	17.53
17.5		0.10	0.54		3.12	15.34
18.5		0.07	0.38		2.82	13.34
19.5		0.00	0.27		0.00	11.53
20.5			0.19			9.89
21.5			0.13			8.40
22.5			0.09			7.05
23.5			0.06			5.84
24.5			0.04			4.76
25.5			0.03			3.81
26.5			0.02			3.00
27.5			0.01			2.30
28.5			0.00			0.00